Estimating Intergenerational and Assortative Processes in Extended Family Data:

Additional Derivations

I The General Model

We assume that the value of the outcome y for an individual from generation t is given by

$$y_t^k = \beta^k \widetilde{y}_{t-1}^k + z_t^k + x_t^k + u_t^k \tag{I.1}$$

where the superscript $k = m$ stands for males and $k = f$ for females. We assume that

$$\widetilde{y}_{t-1}^k = \alpha_y^k y_{t-1}^m + (1 - \alpha_y^k) y_{t-1}^f$$

and the socioeconomic status of the child, z_t^k, depends on the father z_{t-1}^m as well as on the mother z_{t-1}^f

$$\begin{aligned} z_t^k &= \gamma^k \widetilde{z}_{t-1}^k + e_t^k + v_t^k \\ \widetilde{z}_{t-1}^k &= \alpha_z^k z_{t-1}^m + (1 - \alpha_z^k) z_{t-1}^f \end{aligned} \tag{I.2}$$

Regarding the shocks, we assume that x_t^k and e_t^k are shared by all siblings of the same gender, can be correlated across siblings of different gender and are uncorrelated to each other and with the other variables (in particular with z_{t-1}^k and y_{t-1}^l, $l = m, f$). Finally u_t^k and v_t^k are white-noise errors.

We assume throughout the appendix that the economy is in the steady state, and therefore all the parameters and the moments of all the variables are time invariant.

I.1 Assortative mating process

We assume there is assortative mating both in years of schooling and in socioeconomic status (see Behrman and Rosenzweig, 2002, for a related model with assortative mating in two dimensions). In particular we consider the linear projections of z_{t-1}^f and y_{t-1}^f on z_{t-1}^m and y_{t-1}^m:

$$\begin{aligned} z_{t-1}^f &= r_{zz}^m z_{t-1}^m + r_{zy}^m y_{t-1}^m + w_{t-1}^m \\ y_{t-1}^f &= r_{yz}^m z_{t-1}^m + r_{yy}^m y_{t-1}^m + \varepsilon_{t-1}^m \end{aligned}$$

where w_{t-1}^m and ε_{t-1}^m might be correlated but are uncorrelated with z_{t-1}^m and y_{t-1}^m.

The coefficients of the linear projections depend on the correlations $\rho_{z^m y^m}$, $\rho_{z^m z^f}$, $\rho_{z^m y^f}$, $\rho_{y^m z^f}$ and $\rho_{y^m y^f}$, as well as on the standard deviations of z_{t-1}^k and y_{t-1}^k, $k = m, f$:

$$r_{zz}^m = \frac{1}{(1 - \rho_{z^m y^m}^2)} \frac{\sigma_{z^f}}{\sigma_{z^m}} (\rho_{z^m z^f} - \rho_{z^m y^m} \rho_{y^m z^f})$$

$$r_{zy}^m = \frac{1}{(1 - \rho_{z^m y^m}^2)} \frac{\sigma_{z^f}}{\sigma_{y^m}} (\rho_{y^m z^f} - \rho_{z^m y^m} \rho_{z^m z^f})$$

$$r_{yz}^m = \frac{1}{(1 - \rho_{z^m y^m}^2)} \frac{\sigma_{y^f}}{\sigma_{z^m}} (\rho_{z^m y^f} - \rho_{z^m y^m} \rho_{y^m y^f})$$

$$r^m_{yy} = \frac{1}{(1-\rho^2_{z^m y^m})} \frac{\sigma_{y^f}}{\sigma_{y^m}} (\rho_{y^m y^f} - \rho_{z^m y^m}\rho_{z^m y^f})$$

We use these matching functions to write years of schooling, y^k_t, and social status, z^k_t, as a function of father's years of schooling, y^m_{t-1}, and social status z^m_{t-1}. We can write (I.2) as

$$z^k_t = G^k_{zm} z^m_{t-1} + G^k_{ym} y^m_{t-1} + g^k_m \omega^m_{t-1} + e^k_t + v^k_t$$

where

$$\begin{aligned} G^k_{zm} &= \gamma^k(\alpha^k_z + (1-\alpha^k_z) r^m_{zz}) \\ G^k_{ym} &= \gamma^k(1-\alpha^k_z) r^m_{zy} \\ g^k_m &= \gamma^k(1-\alpha^k_z) \end{aligned}$$

and (I.1) as

$$y^k_t = B^k_{ym} y^m_{t-1} + B^k_{zm} z^m_{t-1} + b^k_m \varepsilon^m_{t-1} + g^k_m \omega^m_{t-1} + e^k_t + v^k_t + x^k_t + u^k_t$$

where

$$\begin{aligned} B^k_{ym} &= \beta^k\left(\alpha^k_y + (1-\alpha^k_y) r^m_{yy}\right) + G^k_{ym} \\ B^k_{zm} &= \beta^k(1-\alpha^k_y) r^m_{yz} + G^k_{zm} \\ b^k_m &= \beta^k(1-\alpha^k_y) \end{aligned}$$

All these expressions will be used to compute correlations between relatives that are related through their fathers. However, when we consider relatives that are related through their mothers, we need to find expressions for y^k_t and z^k_t as functions of mother's years of schooling, y^f_{t-1}, and social status z^f_{t-1}.We then also consider the linear projections of z^m_{t-1} and y^m_{t-1} on z^f_{t-1} and y^f_{t-1}:

$$\begin{aligned} z^m_{t-1} &= r^f_{zz} z^f_{t-1} + r^f_{zy} y^f_{t-1} + w^f_{t-1} \\ y^m_{t-1} &= r^f_{yz} z^f_{t-1} + r^f_{yy} y^f_{t-1} + \varepsilon^f_{t-1} \end{aligned}$$

where w^f_{t-1} and ε^f_{t-1} might be correlated but are uncorrelated with z^f_{t-1} and y^f_{t-1}.

The coefficients of the linear projections depend on $\rho_{z^f y^f}, \rho_{z^m z^f}$, $\rho_{z^m y^f}$, $\rho_{y^m z^f}$ and $\rho_{y^m y^f}$, as well as on the standard deviations of z^k_{t-1} and y^k_{t-1}, $k = m, f$:

$$r^f_{zz} = \frac{1}{(1-\rho^2_{z^f y^f})} \frac{\sigma_{z^m}}{\sigma_{z^f}} (\rho_{z^m z^f} - \rho_{z^f y^f}\rho_{z^m y^f})$$

$$r^f_{zy} = \frac{1}{(1-\rho^2_{z^f y^f})} \frac{\sigma_{z^m}}{\sigma_{y^f}} (\rho_{z^m y^f} - \rho_{z^f y^f}\rho_{z^m z^f})$$

$$r_{yz}^f = \frac{1}{(1-\rho_{z^f y^f}^2)} \frac{\sigma_{y^m}}{\sigma_{z^f}} (\rho_{y^m z^f} - \rho_{z^f y^f} \rho_{y^m y^f})$$

$$r_{yy}^f = \frac{1}{(1-\rho_{z^f y^f}^2)} \frac{\sigma_{y^m}}{\sigma_{y^f}} (\rho_{y^m y^f} - \rho_{z^f y^f} \rho_{y^m z^f})$$

Using these linear projections, we can write (I.2) as

$$z_t^k = G_{zf}^k z_{t-1}^f + G_{yf}^k y_{t-1}^f + g_f^k \omega_{t-1}^f + e_t^k + v_t^k$$

where

$$\begin{aligned} G_{zf}^k &= \gamma^k(\alpha_z^k r_{zz}^f + (1-\alpha_z^k)) \\ G_{yf}^k &= \gamma^k \alpha_z^k r_{zy}^f \\ g_f^k &= \gamma^k \alpha_z^k \end{aligned}$$

and (I.1) as

$$y_t^k = B_{yf}^k y_{t-1}^f + B_{zf}^k z_{t-1}^f + b_f^k \varepsilon_{t-1}^f + g_f^k \omega_{t-1}^f + e_t^k + v_t^k + x_t^k + u_t^k$$

where

$$\begin{aligned} B_{yf}^k &= \beta^k \left(\alpha_y^k r_{yy}^f + (1-\alpha_y^k)\right) + G_{yf}^k \\ B_{zf}^k &= \beta^k \alpha_y^k r_{yz}^f + G_{zf}^k \\ b_f^k &= \beta^k \alpha_y^k \end{aligned}$$

I.2 Steady state assumption

As mentioned above, we assume that the second order moments of all variables are time invariant. This steady state assumption implies that $\rho_{z^m y^m}$ and $\rho_{z^f y^f}$ depend on the remaining parameters of the model as shown below.

$$\begin{aligned} Cov(y_t^m, z_t^m) &= Cov(\beta^m \widetilde{y}_{t-1}^m + z_t^m, z_t^m) = Cov(\beta^m(\alpha_y^m y_{t-1}^m + (1-\alpha_y^m) y_{t-1}^f), \gamma^m(\alpha_z^m z_{t-1}^m + (1-\alpha_z^m) z_{t-1}^f)) + \sigma_{z^m}^2 \\ &= \beta^m \alpha_y^m \gamma^m \alpha_z^m Cov(y_{t-1}^m, z_{t-1}^m) + \beta^m \alpha_y^m \gamma^m (1-\alpha_z^m) Cov(y_{t-1}^m, z_{t-1}^f) \\ &+ \beta^m (1-\alpha_y^m) \gamma^m \alpha_z^m Cov(y_{t-1}^f, z_{t-1}^m) + \beta^m (1-\alpha_y^m) \gamma^m (1-\alpha_z^m) Cov(y_{t-1}^f, z_{t-1}^f) + \sigma_{z^m}^2 \end{aligned}$$

Dividing by σ_{z^m} and σ_{y^m}, we have

$$\begin{aligned} \rho_{z^m y^m} &= \beta^m \alpha_y^m \gamma^m \alpha_z^m \rho_{z^m y^m} + \beta^m \alpha_y^m \gamma^m (1-\alpha_z^m) \frac{\sigma_{z^f}}{\sigma_{z^m}} \rho_{y^m z^f} \\ &+ \beta^m (1-\alpha_y^m) \gamma^m \alpha_z^m \frac{\sigma_{y^f}}{\sigma_{y^m}} \rho_{z^m y^f} + \beta^m (1-\alpha_y^m) \gamma^m (1-\alpha_z^m) \frac{\sigma_{z^f}}{\sigma_{z^m}} \frac{\sigma_{y^f}}{\sigma_{y^m}} \rho_{z^f y^f} + \frac{\sigma_{z^m}}{\sigma_{y^m}} \end{aligned}$$

and rearranging

$$
\begin{aligned}
& (1-\beta^m\alpha_y^m\gamma^m\alpha_z^m)\rho_{z^my^m} - \beta^m(1-\alpha_y^m)\gamma^m(1-\alpha_z^m)\frac{\sigma_{z^f}}{\sigma_{z^m}}\frac{\sigma_{y^f}}{\sigma_{y^m}}\rho_{z^fy^f} \\
= \ & \frac{\sigma_{z^m}}{\sigma_{y^m}} + \beta^m\alpha_y^m\gamma^m(1-\alpha_z^m)\frac{\sigma_{z^f}}{\sigma_{z^m}}\rho_{y^mz^f} + \beta^m(1-\alpha_y^m)\gamma^m\alpha_z^m\frac{\sigma_{y^f}}{\sigma_{y^m}}\rho_{z^my^f}
\end{aligned}
$$

analogously

$$
\begin{aligned}
& -\beta^f(1-\alpha_y^f)\gamma^f(1-\alpha_z^f)\frac{\sigma_{z^m}}{\sigma_{z^f}}\frac{\sigma_{y^m}}{\sigma_{y^f}}\rho_{z^my^m} + (1-\beta^f\alpha_y^f\gamma^f\alpha_z^f)\rho_{z^fy^f} \\
= \ & \frac{\sigma_{z^f}}{\sigma_{y^f}} + \beta^f\alpha_y^f\gamma^f(1-\alpha_z^f)\frac{\sigma_{z^m}}{\sigma_{z^f}}\rho_{z^my^f} + \beta^f(1-\alpha_y^f)\gamma^f\alpha_z^f\frac{\sigma_{y^m}}{\sigma_{y^f}}\rho_{y^mz^f}
\end{aligned}
$$

and from these two equations we can solve for $\rho_{z^my^m}$ and $\rho_{z^fy^f}$ as a function of $\rho_{y^mz^f}$, $\rho_{z^my^f}$ and some other parameters of the model.

We then have that the model has 20 parameters $\gamma^k, \beta^k, \alpha_z^k, \alpha_y^k, \sigma_{z^k}^2, \sigma_{x^k}^2, \sigma_{e^k}^2,\ k = m, f,$ and $\sigma_{x^mx^f}, \sigma_{e^me^f}$, $\rho_{z^mz^f}, \rho_{y^mz^f}, \rho_{z^my^f}$ and $\rho_{y^my^f}$

I.3 Covariances

I.3.1 Main covariances

We use the notation in Figure 1 to denote individuals with different degrees of kinship. We first compute the main covariances (husband-wife, parent-child and siblings). Then, the covariances for other relatives are obtained recursively.

Husband and wife $a - a'$

We have to compute the covariance between "a" and "a'". Let $n' = m, f$ be the gender of a' and $n = m, f$ the gender of the a.

$$
Cov(y_{t-1}^{a,n}, y_{t-1}^{a',n'}) = \sigma_{y^m}\sigma_{y^f}\rho_{y^my^f}
$$

Parent–child $aa - a'$

We have to compute the covariances between "aa" and "a'". Let $n' = m, f$ be the gender of a' and $n^* = m, f$ the gender of the aa. We project aa on a' (his/her father or mother) who has gender n'

$$
\begin{aligned}
Cov(z_t^{aa,n^*}, z_{t-1}^{a',n'}) &= Cov(G_{yn'}^{n^*}y_{t-1}^{n'} + G_{zn'}^{n^*}z_{t-1}^{n'}, z_{t-1}^{n'}) \\
&= G_{yn'}^{n^*}\sigma_{z^{n'}}\sigma_{y^{n'}}\rho_{z^{n'}y^{n'}} + G_{zn'}^{n^*}\sigma_{z^{n'}}^2
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{aa,n^*}, y_{t-1}^{a',n'}) &= Cov(G_{yn'}^{n^*}y_{t-1}^{n'} + G_{zn'}^{n^*}z_{t-1}^{n'}, y_{t-1}^{n'}) \\
&= G_{yn'}^{n^*}\sigma_{y^{n'}}^2 + G_{zn'}^{n^*}\sigma_{z^{n'}}\sigma_{y^{n'}}\rho_{z^{n'}y^{n'}}
\end{aligned}
$$

$$
\begin{aligned}
Cov(y_t^{aa,n^*}, z_{t-1}^{a',n'}) &= Cov(B_{yn'}^{n^*} y_{t-1}^{n'} + B_{zn'}^{n^*} z_{t-1}^{n'}, z_{t-1}^{n'}) \\
&= B_{yn'}^{n^*} \sigma_{z^{n'}} \sigma_{y^{n'}} \rho_{z^{n'} y^{n'}} + B_{zn'}^{k} \sigma_{z^{n'}}^2
\end{aligned}
$$

$$
\begin{aligned}
Cov(y_t^{aa,n^*}, y_{t-1}^{a',n'}) &= Cov(B_{yn'}^{n^*} y_{t-1}^{n'} + B_{zn'}^{n^*} z_{t-1}^{n'}, y_{t-1}^{n'}) \\
&= B_{yn'}^{n^*} \sigma_{y^{n'}}^2 + B_{zn'}^{n^*} \sigma_{z^{n'}} \sigma_{y^{n'}} \rho_{z^{n'} y^{n'}}
\end{aligned}
$$

Siblings $a' - b$

We have to compute the covariances between "a'" and "b". Let $n' = m, f$ be the gender of a' and $l = m, f$ the gender of the b. We project a' and b on their father or mother $GP2$ who has gender k

$$
\begin{aligned}
&Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l}) = Cov(G_{yk}^{n'} y_{t-2}^{GP2,k} + G_{zk}^{n'} z_{t-2}^{GP2,k} + g_k^{n'} \omega_{t-2}^k + e_{t-1}^{a',n'}, G_{yk}^{l} y_{t-2}^{GP2,k} + G_{zk}^{l} z_{t-2}^{GP2,k} + g_k^{l} \omega_{t-2}^k + e_{t-1}^{b,l}) \\
&= G_{yk}^{n'} G_{yk}^{l} \sigma_{y^k}^2 + G_{zk}^{n'} G_{zk}^{l} \sigma_{z^k}^2 + \left(G_{yk}^{n'} G_{zk}^{l} + G_{zk}^{n'} G_{yk}^{l} \right) \sigma_{z^{n'}} \sigma_{y^{n'}} \rho_{z^{n'} y^{n'}} \\
&+ g_k^{n'} g_k^{l} \sigma_{w^k}^2 + \sigma_{e^{n'} e^{l}}
\end{aligned}
$$

$$
\begin{aligned}
&Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l}) = Cov(G_{yk}^{n'} y_{t-2}^{GP2,k} + G_{zk}^{n'} z_{t-2}^{GP2,k} + g_k^{n'} \omega_{t-2}^k + e_{t-1}^{a',n'}, B_{yk}^{l} y_{t-2}^{GP2,k} + B_{zk}^{l} z_{t-2}^{GP2,k} + b_k^{l} \varepsilon_{t-2}^k + g_k^{l} \omega_{t-2}^k + e_{t-1}^{b,l}) \\
&= G_{yk}^{n'} B_{yk}^{l} \sigma_{y^k}^2 + G_{zk}^{n'} B_{zk}^{l} \sigma_{z^k}^2 + \left(G_{yk}^{n'} B_{zk}^{l} + G_{zk}^{n'} B_{yk}^{l} \right) \sigma_{z^{n'}} \sigma_{y^{n'}} \rho_{z^{n'} y^{n'}} \\
&+ g_k^{n'} g_k^{l} \sigma_{w^k}^2 + \sigma_{e^{n'} e^{l}} + g_k^{n'} b_k^{l} Cov(\varepsilon_{t-1}^k, \omega_{t-1}^k)
\end{aligned}
$$

$$
\begin{aligned}
&Cov(y_{t-1}^{a',n'}, z_{t-1}^{b,l}) = Cov(B_{yk}^{n'} y_{t-2}^{GP2,k} + B_{zk}^{n'} z_{t-2}^{GP2,k} + b_k^{n'} \varepsilon_{t-2}^{n'} + g_k^{n'} \omega_{t-2}^k + e_{t-1}^{a',n'}, G_{yk}^{l} y_{t-2}^{GP2,k} + G_{zk}^{l} z_{t-2}^{GP2,k} + g_k^{l} \omega_{t-2}^k + e_{t-1}^{b,l}) \\
&= B_{yk}^{n'} G_{yk}^{l} \sigma_{y^k}^2 + B_{zk}^{n'} G_{zk}^{l} \sigma_{z^k}^2 + \left(B_{yk}^{n'} G_{zk}^{l} + B_{zk}^{n'} G_{yk}^{l} \right) \sigma_{z^{n'}} \sigma_{y^{n'}} \rho_{z^{n'} y^{n'}} \\
&+ g_k^{n'} g_k^{l} \sigma_{w^k}^2 + \sigma_{e^{n'} e^{l}} + b_k^{n'} g_k^{l} Cov(\varepsilon_{t-1}^k, \omega_{t-1}^k)
\end{aligned}
$$

$$
\begin{aligned}
&Cov(y_{t-1}^{a',n'}, y_{t-1}^{b,l}) = Cov(B_{yk}^{n'} y_{t-2}^{GP2,k} + B_{zk}^{n'} z_{t-2}^{GP2,k} + b_k^{n'} \varepsilon_{t-2}^{n'} + g_k^{n'} \omega_{t-2}^k + e_{t-1}^{a',n'} + x_{t-1}^{a',n'}, B_{yk}^{l} y_{t-2}^{GP2,k} + B_{zk}^{l} z_{t-2}^{GP2,k} + b_k^{l} \varepsilon_{t-2}^k + g_k^{l} \omega_{t-2}^k + e_{t-1}^{b,l} + x_{t-1}^{b,l}) \\
&= B_{yk}^{n'} B_{yk}^{l} \sigma_{y^k}^2 + B_{zk}^{n'} B_{zk}^{l} \sigma_{z^k}^2 + \left(B_{yk}^{n'} B_{zk}^{l} + B_{zk}^{n'} B_{yk}^{l} \right) \sigma_{z^{n'}} \sigma_{y^{n'}} \rho_{z^{n'} y^{n'}} \\
&+ g_k^{n'} g_k^{l} \sigma_{w^k}^2 + b_k^{n'} b_k^{l} \sigma_{\varepsilon^k}^2 + \sigma_{e^{n'} e^{l}} + \left(b_k^{n'} g_k^{l} + g_k^{n'} b_k^{l} \right) Cov(\varepsilon_{t-1}^{n'}, \omega_{t-1}^{n'}) + \sigma_{x^{n'} x^{l}}
\end{aligned}
$$

I.3.2 Other covariances

Before we obtain the remaining covariances for different degrees of kinship we compute the linear projections of $z_{t-1}^{a',n'}$ and $y_{t-1}^{a',n'}$ on $z_{t-1}^{b,l}$ and $y_{t-1}^{b,l}$, $n', l = m, f$, where a' and b are siblings

$$
\begin{aligned}
z_{t-1}^{a',n'} &= r_{zz}^{n',l} z_{t-1}^{b,l} + r_{zy}^{n',l} y_{t-1}^{b,l} + w_{t-1}^{b,l} \\
y_{t-1}^{a',n'} &= r_{yz}^{n',l} z_{t-1}^{b,l} + r_{yy}^{n',l} y_{t-1}^{b,l} + \varepsilon_{t-1}^{b,l}
\end{aligned}
$$

where $w_{t-1}^{b,l}$ and $\varepsilon_{t-1}^{b,l}$ might be correlated but are uncorrelated with $z_{t-1}^{b,l}$ and $y_{t-1}^{b,l}$ and

$$
r_{zz}^{n',l} = \frac{1}{\sigma_{z^l}^2 \sigma_{y^l}^2 - \sigma_{z^l y^l}^2} \left(\sigma_{y^l}^2 \sigma_{z^{a',n'} z^{b,l}} - \sigma_{z^l y^l} \sigma_{z^{a',n'} y^{b,l}} \right)
$$

$$r_{zy}^{n',l} = \frac{1}{\sigma_{z^l}^2 \sigma_{y^l}^2 - \sigma_{z^l y^l}^2} \left(\sigma_{z^l}^2 \sigma_{z^{a',n'} y^{b,l}} - \sigma_{z^l y^l} \sigma_{z^{a',n'} z^{b,l}} \right)$$

$$r_{yz}^{n',l} = \frac{1}{\sigma_{z^l}^2 \sigma_{y^l}^2 - \sigma_{z^l y^l}^2} \left(\sigma_{y^l}^2 \sigma_{y^{a',n'} z^{b,l}} - \sigma_{z^l y^l} \sigma_{y^{a',n'} y^{b,l}} \right)$$

$$r_{yy}^{n',l} = \frac{1}{\sigma_{z^l}^2 \sigma_{y^l}^2 - \sigma_{z^l y^l}^2} \left(\sigma_{z^l}^2 \sigma_{y^{a',n'} y^{b,l}} - \sigma_{z^l y^l} \sigma_{y^{a',n'} z^{b,l}} \right)$$

Notice that error terms, $w_{t-1}^{b,l}$ and $\varepsilon_{t-1}^{b,l}$, are likely to be correlated with the latent factor and the outcome of a, the spouse of a', and also with error terms of the linear projections of a' on a. However, since we use these linear projections to find the correlation with in-law relatives of b, what is relevant is whether $w_{t-1}^{b,l}$ and $\varepsilon_{t-1}^{b,l}$ are correlated with $z_{t-1}^{b',l}$ and $y_{t-1}^{b',l}$, where b' is the spouse of b. Since these error terms are not correlated with $z_{t-1}^{b,l}$ and $y_{t-1}^{b,l}$, and we are assuming that the assortative mating is in z and y, they are not correlated with $z_{t-1}^{b',l}$ and $y_{t-1}^{b',l}$ either.

Consanguine relatives (”blood”)

Vertical covariances

Uncle/aunt (siblings of the parents) $aa - b$

We have to compute the covariances between ”aa” and ”b”. Let $n^* = m, f$ be the gender of aa and $l = m, f$ the gender of the b. We project aa on a' (his/her father or mother) who has gender n'

$$Cov(z_t^{aa,n^*}, z_{t-1}^{b,l}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'} + G_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{b,l}) = G_{zn'}^{n^*} Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l}) + G_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(z_t^{aa,n^*}, y_{t-1}^{b,l}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'} + G_{yn'}^{n^*} y_{t-1}^{a',n'}, y_{t-1}^{b,l}) = G_{zn'}^{n^*} Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l}) + G_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, y_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, z_{t-1}^{b,l}) = Cov(B_{zn'}^{n^*} z_{t-1}^{a',n'} + B_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{b,l}) = B_{zn'}^{n^*} Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l}) + B_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, y_{t-1}^{b,l}) = Cov(B_{zn'}^{n^*} z_{t-1}^{a',n'} + B_{yn'}^{n^*} y_{t-1}^{a',n'}, y_{t-1}^{b,l}) = B_{zn'}^{n^*} Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l}) + B_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, y_{t-1}^{b,l})$$

where a' and b are siblings.

Horizontal covariances

Cousins $aa - bb$

We have to compute the covariances between ”aa” and ”bb”. Let $n^* = m, f$ be the gender of aa and $l^* = m, f$ the gender of the bb. We project bb on b (his/her father or mother) who has gender l

$$Cov(z_t^{aa,n^*}, z_t^{bb,l^*}) = Cov(z_t^{aa,n^*}, G_{zl}^{l^*} z_{t-1}^{b,l} + G_{yl}^{l^*} y_{t-1}^{b,l}) = G_{zl}^{l^*} Cov(z_t^{aa,n^*}, z_{t-1}^{b,l}) + G_{yl}^{l^*} Cov(z_t^{aa,n^*}, y_{t-1}^{b,l})$$

$$Cov(z_t^{aa,n^*}, y_t^{bb,l^*}) = Cov(z_t^{aa,n^*}, B_{zl}^{l^*} z_{t-1}^{b,l} + B_{yl}^{l^*} y_{t-1}^{b,l}) = B_{zl}^{l^*} Cov(z_t^{aa,n^*}, z_{t-1}^{b,l}) + B_{yl}^{l^*} Cov(z_t^{aa,n^*}, y_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, z_t^{bb,l^*}) = Cov(y_t^{aa,n^*}, G_{zl}^{l^*} z_{t-1}^{b,l} + G_{yl}^{l^*} y_{t-1}^{b,l}) = G_{zl}^{l^*} Cov(y_t^{aa,n^*}, z_{t-1}^{b,l}) + G_{yl}^{l^*} Cov(y_t^{aa,n^*}, y_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, y_t^{bb,l^*}) = Cov(y_t^{aa,n^*}, B_{zl}^{l^*} z_{t-1}^{b,l} + B_{yl}^{l^*} y_{t-1}^{b,l}) = B_{zl}^{l^*} Cov(y_t^{aa,n^*}, z_{t-1}^{b,l}) + B_{yl}^{l^*} Cov(y_t^{aa,n^*}, y_{t-1}^{b,l})$$

where b is the uncle/aunt of aa.

Affinity relatives (”in-law”)

Vertical covariances

Spouse of the uncle/aunt (spouses of the siblings of the parents) $aa - b'$

We have to compute the covariances between ”aa” and ”b'”. Let $n^* = m, f$ be the gender of aa and $l' = m, f$ the gender of the b'. We project b' on his/her spouse b who has gender l

$$Cov(z_t^{aa,n^*}, z_{t-1}^{b',l'}) = Cov(z_t^{aa,n^*}, r_{zz}^{l} z_{t-1}^{b,l} + r_{zy}^{l} y_{t-1}^{b,l}) = r_{zz}^{l} Cov(z_t^{aa,n^*}, z_{t-1}^{b,l}) + r_{zy}^{l} Cov(z_t^{aa,n^*}, y_{t-1}^{b,l})$$

$$Cov(z_t^{aa,n^*}, y_{t-1}^{b',l'}) = Cov(z_t^{aa,n^*}, r_{yz}^{l} z_{t-1}^{b,l} + r_{yy}^{l} y_{t-1}^{b,l}) = r_{yz}^{l} Cov(z_t^{aa,n^*}, z_{t-1}^{b,l}) + r_{yy}^{l} Cov(z_t^{aa,n^*}, y_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, z_{t-1}^{b',l'}) = Cov(y_t^{aa,n^*}, r_{zz}^{l} z_{t-1}^{b,l} + r_{zy}^{l} y_{t-1}^{b,l}) = r_{zz}^{l} Cov(y_t^{aa,n^*}, z_{t-1}^{b,l}) + r_{zy}^{l} Cov(y_t^{aa,n^*}, y_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, y_{t-1}^{b',l'}) = Cov(y_t^{aa,n^*}, r_{yz}^{l} z_{t-1}^{b,l} + r_{yy}^{l} y_{t-1}^{b,l}) = r_{yz}^{l} Cov(y_t^{aa,n^*}, z_{t-1}^{b,l}) + r_{yy}^{l} Cov(y_t^{aa,n^*}, y_{t-1}^{b,l})$$

where b is uncle/aunt of aa.

Siblings of the siblings in law of the parents $aa - c$

We have to compute the covariances between ”aa” and ”c”. Let $n^* = m, f$ be the gender of aa and $o = m, f$ the gender of the c. We project c on his/her sibling b' who has gender l'

$$Cov(z_t^{aa,n^*}, z_{t-1}^{c,o}) = Cov(z_t^{aa,n^*}, r_{zz}^{o,l'} z_{t-1}^{b',l'} + r_{zy}^{o,l'} y_{t-1}^{b',l'}) = r_{zz}^{o,l'} Cov(z_t^{aa,n^*}, z_{t-1}^{b',l'}) + r_{zy}^{o,l'} Cov(z_t^{aa,n^*}, y_{t-1}^{b',l'})$$

$$Cov(z_t^{aa,n^*}, y_{t-1}^{c,o}) = Cov(z_t^{aa,n^*}, r_{yz}^{o,l'} z_{t-1}^{b',l'} + r_{yy}^{o,l'} y_{t-1}^{b',l'}) = r_{yz}^{o,l'} Cov(z_t^{aa,n^*}, z_{t-1}^{b',l'}) + r_{yy}^{o,l'} Cov(z_t^{aa,n^*}, y_{t-1}^{b',l'})$$

$$Cov(y_t^{aa,n^*}, z_{t-1}^{c,o}) = Cov(y_t^{aa,n^*}, r_{zz}^{o,l'} z_{t-1}^{b',l'} + r_{zy}^{o,l'} y_{t-1}^{b',l'}) = r_{zz}^{o,l'} Cov(y_t^{aa,n^*}, z_{t-1}^{b',l'}) + r_{zy}^{o,l'} Cov(y_t^{aa,n^*}, y_{t-1}^{b',l'})$$

$$Cov(y_t^{aa,n^*}, y_{t-1}^{c,o}) = Cov(y_t^{aa,n^*}, r_{yz}^{o,l'} z_{t-1}^{b',l'} + r_{yy}^{o,l'} y_{t-1}^{b',l'}) = r_{yz}^{o,l'} Cov(y_t^{aa,n^*}, z_{t-1}^{b',l'}) + r_{yy}^{o,l'} Cov(y_t^{aa,n^*}, y_{t-1}^{b',l'})$$

where b' is the spouse of the uncle/aunt of aa.

We can recursively compute the covariances for the spouses of the siblings in law of the parents and the siblings of the siblings in law of the parents of any degree.

Horizontal covariances

Siblings in law $a - b$

We have to compute the covariances between ”a” and ”b”. Let $n = m, f$ be the gender of a and $l = m, f$ the gender of the b. We project a on his/her spouse a' who has gender n'

$$Cov(z_{t-1}^{a,n}, z_{t-1}^{b,l}) = Cov(r_{zz}^{n'} z_{t-1}^{a',n'} + r_{zy}^{n'} y_{t-1}^{a',n'}, z_{t-1}^{b,l}) = r_{zz}^{n'} Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l}) + r_{zy}^{n'} Cov(y_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(z_{t-1}^{a,n}, y_{t-1}^{b,l}) = Cov(r_{zz}^{n'} z_{t-1}^{a',n'} + r_{zy}^{n'} y_{t-1}^{a',n'}, y_{t-1}^{b,l}) = r_{zz}^{n'} Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l}) + r_{zy}^{n'} Cov(y_{t-1}^{a',n'}, y_{t-1}^{b,l})$$

$$Cov(y_{t-1}^{a,n}, z_{t-1}^{b,l}) = Cov(r_{yz}^{n'} z_{t-1}^{a',n'} + r_{yy}^{n'} y_{t-1}^{a',n'}, z_{t-1}^{b,l}) = r_{yz}^{n'} Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l}) + r_{yy}^{n'} Cov(y_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(y_{t-1}^{a,n}, y_{t-1}^{b,l}) = Cov(r_{yz}^{n'} z_{t-1}^{a',n'} + r_{yy}^{n'} y_{t-1}^{a',n'}, y_{t-1}^{b,l}) = r_{yz}^{n'} Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l}) + r_{yy}^{n'} Cov(y_{t-1}^{a',n'}, y_{t-1}^{b,l})$$

where a' and b are siblings.

Spouse of the siblings in law $a - b'$

We have to compute the covariances between "a" and "b'". Let $n = m, f$ be the gender of a and $l' = m, f$ the gender of the b'. We project b' on his/her spouse b

$$Cov(z_{t-1}^{a,n}, z_{t-1}^{b',l'}) = Cov(z_{t-1}^{a,n}, r_{zz}^{l} z_{t-1}^{b,l} + r_{zy}^{l} y_{t-1}^{b,l}) = r_{zz}^{l} Cov(z_{t-1}^{a,n}, z_{t-1}^{b,l}) + r_{zy}^{l} Cov(z_{t-1}^{a,n}, y_{t-1}^{b,l})$$

$$Cov(z_{t-1}^{a,n}, y_{t-1}^{b',l'}) = Cov(z_{t-1}^{a,n}, r_{yz}^{l} z_{t-1}^{b,l} + r_{yy}^{l} y_{t-1}^{b,l}) = r_{yz}^{l} Cov(z_{t-1}^{a,n}, z_{t-1}^{b,l}) + r_{yy}^{l} Cov(z_{t-1}^{a,n}, y_{t-1}^{b,l})$$

$$Cov(y_{t-1}^{a,n}, z_{t-1}^{b',l'}) = Cov(y_{t-1}^{a,n}, r_{zz}^{l} z_{t-1}^{b,l} + r_{zy}^{l} y_{t-1}^{b,l}) = r_{zz}^{l} Cov(y_{t-1}^{a,n}, z_{t-1}^{b,l}) + r_{zy}^{l} Cov(y_{t-1}^{a,n}, y_{t-1}^{b,l})$$

$$Cov(y_{t-1}^{a,n}, y_{t-1}^{b',l'}) = Cov(y_{t-1}^{a,n}, r_{yz}^{l} z_{t-1}^{b,l} + r_{yy}^{l} y_{t-1}^{b,l}) = r_{yz}^{l} Cov(y_{t-1}^{a,n}, z_{t-1}^{b,l}) + r_{yy}^{l} Cov(y_{t-1}^{a,n}, y_{t-1}^{b,l})$$

where a and b are siblings in law.

Sibling of the sibling in law

We have to compute the covariances between "a'" and "c". Let $n' = m, f$ be the gender of a' and $o = m, f$ the gender of the c. We project a' on his/her sibling b who has gender l

$$Cov(z_{t-1}^{a',n'}, z_{t-1}^{c,o}) = Cov(r_{zz}^{n',l} z_{t-1}^{b,l} + r_{zy}^{n',l} y_{t-1}^{b,l}, z_{t-1}^{c,o}) = r_{zz}^{n',l} Cov(z_{t-1}^{b,l}, z_{t-1}^{c,o}) + r_{zy}^{n',l} Cov(y_{t-1}^{b,l}, z_{t-1}^{c,o})$$

$$Cov(z_{t-1}^{a',n'}, y_{t-1}^{c,o}) = Cov(r_{zz}^{n',l} z_{t-1}^{b,l} + r_{zy}^{n',l} y_{t-1}^{b,l}, y_{t-1}^{c,o}) = r_{zz}^{n',l} Cov(z_{t-1}^{b,l}, y_{t-1}^{c,o}) + r_{zy}^{n',l} Cov(y_{t-1}^{b,l}, y_{t-1}^{c,o})$$

$$Cov(y_{t-1}^{a',n'}, z_{t-1}^{c,o}) = Cov(r_{yz}^{n',l} z_{t-1}^{b,l} + r_{yy}^{n',l} y_{t-1}^{b,l}, z_{t-1}^{c,o}) = r_{yz}^{n',l} Cov(z_{t-1}^{b,l}, z_{t-1}^{c,o}) + r_{yy}^{n',l} Cov(y_{t-1}^{b,l}, z_{t-1}^{c,o})$$

$$Cov(y_{t-1}^{a',n'}, y_{t-1}^{c,o}) = Cov(r_{yz}^{n',l} z_{t-1}^{b,l} + r_{yy}^{n',l} y_{t-1}^{b,l}, y_{t-1}^{c,o}) = r_{yz}^{n',l} Cov(z_{t-1}^{b,l}, y_{t-1}^{c,o}) + r_{yy}^{n',l} Cov(y_{t-1}^{b,l}, y_{t-1}^{c,o})$$

where b and c are siblings in law.

We can recursively compute the covariances for siblings in law, spouses of the siblings in law and siblings of the siblings in law of any degree.

Cousins in law

We have to compute the covariances between "aa" and "cc". Let $n^* = m, f$ be the gender of aa and $o^* = m, f$

the gender of the cc. We project cc on c (his/her father or mother) who has gender o

$$Cov(z_t^{aa,n^*}, z_t^{cc,o^*}) = Cov(z_t^{aa,n^*}, G_{zo}^{o^*} z_{t-1}^{c,o} + G_{yo}^{o^*} y_{t-1}^{c,o}) = G_{zo}^{o^*} Cov(z_t^{aa,n^*}, z_{t-1}^{c,o}) + G_{yo}^{o^*} Cov(z_t^{aa,n^*}, y_{t-1}^{c,o})$$

$$Cov(z_t^{aa,n^*}, y_t^{cc,o^*}) = Cov(z_t^{aa,n^*}, B_{zo}^{o^*} z_{t-1}^{c,o} + B_{yo}^{o^*} y_{t-1}^{c,o}) = B_{zo}^{o^*} Cov(z_t^{aa,n^*}, z_{t-1}^{c,o}) + B_{yo}^{o^*} Cov(z_t^{aa,n^*}, y_{t-1}^{c,o})$$

$$Cov(y_t^{aa,n^*}, z_t^{cc,o^*}) = Cov(y_t^{aa,n^*}, G_{zo}^{o^*} z_{t-1}^{c,o} + G_{yo}^{o^*} y_{t-1}^{c,o}) = G_{zo}^{o^*} Cov(y_t^{aa,n^*}, z_{t-1}^{c,o}) + G_{yo}^{o^*} Cov(y_t^{aa,n^*}, y_{t-1}^{c,o})$$

$$Cov(y_t^{aa,n^*}, y_t^{cc,o^*}) = Cov(y_t^{aa,n^*}, B_{zo}^{o^*} z_{t-1}^{c,o} + B_{yo}^{o^*} y_{t-1}^{c,o}) = B_{zo}^{o^*} Cov(y_t^{aa,n^*}, z_{t-1}^{c,o}) + B_{yo}^{o^*} Cov(y_t^{aa,n^*}, y_{t-1}^{c,o})$$

where c is the sibling in law of the uncle/aunt of aa. We can recursively compute the covariances for cousins in law of any degree.

J No Direct Effect and Assortative Mating Only in z

We next consider a latent factor model with no direct effect ($\beta = 0$) and assortative mating only in z. This model is less general than the previous one but it has the advantage that we can show how the parameters are identified.

We write the outcome y for an individual from generation t as

$$y_t^k = z_t^k + x_t^k + u_t^k \tag{J.1}$$

where the superscript $k = m$ stands for males and $k = f$ for females. We assume that the socioeconomic status of the child, z_t^k, depends on the father z_{t-1}^m as well as on the mother z_{t-1}^f

$$\begin{aligned} z_t^k &= \gamma^k \tilde{z}_{t-1}^k + e_t^k + v_t^k \\ \tilde{z}_{t-1}^k &= \alpha_z^k z_{t-1}^m + (1-\alpha_z^k) z_{t-1}^f \end{aligned} \tag{J.2}$$

Regarding the shocks, we assume that x_t^k and e_t^k are shared by all siblings of the same gender, can be correlated across siblings of different gender and are uncorrelated to each other and with the other variables (in particular with z_{t-1}^k and y_{t-1}^l, $l = m, f$). Finally u_t^k and v_t^k are white-noise errors.

Notice that from (J.1)

$$Cov(y_t^k, z_t^k) = \sigma_{z^k}^2$$

J.1 Assortative mating process

We assume there is assortative mating only in z, i.e we assume that the coefficients of y_{t-1}^m in the linear projections of z_{t-1}^f and y_{t-1}^fon z_{t-1}^m and y_{t-1}^m are zero. This means that we can write z_{t-1}^f as

$$z_{t-1}^f = \frac{\sigma_{z^f}}{\sigma_{z^m}} \rho_{z^m z^f} z_{t-1}^m + w_{t-1}^m$$

where w_{t-1}^m is uncorrelated with z_{t-1}^m and y_{t-1}^m, and

$$y_{t-1}^f = \frac{\sigma_{z^f}}{\sigma_{z^m}} \rho_{z^m z^f} z_{t-1}^m + w_{t-1}^m + x_{t-1}^f + u_{t-1}^f$$

We have that

$$\sigma_{w^m}^2 = \sigma_{z^f}^2 \left(1 - \rho_{z^m z^f}^2\right)$$

We use these matching functions to write years of schooling, y_t^k, and social status, z_t^k, as a function of the father social status z_{t-1}^m. We can write (J.2) as

$$\begin{aligned} z_t^k &= \gamma^k \left(\alpha_z^k z_{t-1}^m + (1-\alpha_z^k) z_{t-1}^f\right) + e_t^k + v_t^k \\ &= \gamma^k \left(\alpha_z^k z_{t-1}^m + (1-\alpha_z^k) \left(\frac{\sigma_{z^f}}{\sigma_{z^m}} \rho_{z^m z^f} z_{t-1}^m + w_{t-1}^m\right)\right) + e_t^k + v_t^k \\ &= G_{zm}^k z_{t-1}^m + g_m^k \omega_{t-1}^m + e_t^k + v_t^k \end{aligned}$$

where

$$\begin{aligned} G_{zm}^k &= \gamma^k (\alpha_z^k + (1-\alpha_z^k) \frac{\sigma_{z^f}}{\sigma_{z^m}} \rho_{z^m z^f}) \\ g_m^k &= \gamma^k (1-\alpha_z^k) \end{aligned}$$

and (J.1) as

$$y_t^k = G_{zm}^k z_{t-1}^m + g_m^k \omega_{t-1}^m + e_t^k + x_t^k + v_t^k + u_t^k$$

All these expressions will be used to compute correlations between relatives that are related through their fathers. However, when we consider relatives that are related through their mothers, we use the expressions for y_t^k and z_t^k as functions of mother

$$z_t^k = G_{zf}^k z_{t-1}^f + g_f^k \omega_{t-1}^f + e_t^k + v_t^k$$

where

$$\begin{aligned} G_{zf}^k &= \gamma^k (\alpha_z^k \frac{\sigma_{z^m}}{\sigma_{z^f}} \rho_{z^m z^f} + (1-\alpha_z^k)) \\ g_f^k &= \gamma^k \alpha_z^k \end{aligned}$$

and (J.1) as

$$y_t^k = G_{zf}^k z_{t-1}^f + g_f^k \omega_{t-1}^f + e_t^k + x_t^k + v_t^k + u_t^k$$

We then have that the model has 13 parameters $\gamma^k, \alpha_z^k, \sigma_{z^k}^2, \sigma_{x^k}^2, \sigma_{e^k}^2,\ k = m, f,\ \sigma_{x^m x^f}, \sigma_{e^m e^f},\ \rho_{z^m z^f}$.

J.2 Covariances

J.2.1 Main covariances

We first compute the main covariances (husband-wife, parent-child and siblings). Then, the covariances for other relatives are obtained recursively. We again use the notation in Figure 1 to denote individuals with different degrees of kinship.

Husband and wife $a - a'$

We have to compute the covariances between "a" and "a'". Let $n' = m, f$ be the gender of a' and $n = f, m$ the gender of a

$$\begin{aligned} Cov(y_{t-1}^{a,n}, y_{t-1}^{a',n'}) &= Cov\left(\frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f} z_{t-1}^{a',n'}, y_{t-1}^{a',n'}\right) \\ &= \frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f} Cov(z_{t-1}^{a',n'}, y_{t-1}^{a',n'}) = \frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f}\sigma_{z^{n'}}^2 = \sigma_{z^f}\sigma_{z^m}\rho_{z^m z^f} \end{aligned}$$

Parent–child $aa - a'$

We have to compute the covariances between "aa" and "a'". Let $n^* = m, f$ be the gender of aa and $n' = m, f$ the gender a'. We project aa on a' (his/her father or mother).

$$Cov(z_t^{aa,n^*}, z_{t-1}^{a',n'}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'}, z_{t-1}^{a',n'}) = G_{zn'}^{n^*}\sigma_{z^{n'}}^2$$

$$Cov(z_t^{aa,n^*}, y_{t-1}^{a',n'}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'}, y_{t-1}^{a',n'}) = G_{zn'}^{n^*}\sigma_{z^{n'}}^2$$

$$Cov(y_t^{aa,n^*}, z_{t-1}^{a',n'}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'}, z_{t-1}^{a',n'}) = G_{zn'}^{n^*}\sigma_{z^{n'}}^2$$

$$Cov(y_t^{aa,n^*}, y_{t-1}^{a',n'}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'}, y_{t-1}^{a',n'}) = G_{zn'}^{n^*}\sigma_{z^{n'}}^2$$

Notice that in this case the four covariances take the same value

Siblings $a' - b$

We have to compute the covariance between "a'" and "b". Let $n' = m, f$ be the gender of a' and $l = m, f$ the gender of the b. We project a' and b on their father or mother $GP2$ who has gender k

$$\begin{aligned} &Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l}) = Cov(G_{zk}^{n'} z_{t-2}^{GP2,k} + g_k^{n'}\omega_{t-2}^{GP2,k} + e_{t-1}^{a',n'}, G_{zk}^{l} z_{t-2}^{GP2,k} + g_k^{l}\omega_{t-2}^{GP2,k} + e_{t-1}^{b,l}) \\ &= G_{zk}^{n'} G_{zk}^{l}\sigma_{z^k}^2 + g_k^{n'} g_k^{l}\sigma_{w^k}^2 + \sigma_{e^{n'}e^l} \end{aligned}$$

$$\begin{aligned} &Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l}) = Cov(G_{zk}^{n'} z_{t-2}^{GP2,k} + g_k^{n'}\omega_{t-2}^{GP2,k} + e_{t-1}^{a',n'}, G_{zk}^{l} z_{t-2}^{GP2,k} + g_k^{l}\omega_{t-2}^{GP2,k} + e_{t-1}^{b,l}) \\ &= G_{zk}^{n'} G_{zk}^{l}\sigma_{z^k}^2 + g_k^{n'} g_k^{l}\sigma_{w^k}^2 + \sigma_{e^{n'}e^l} \end{aligned}$$

$$\begin{aligned} &Cov(y_t^{a',n'}, y_t^{b,l}) = Cov(G_{zk}^{n'} z_{t-2}^{GP2,k} + g_k^{n'}\omega_{t-2}^{GP2,k} + e_{t-1}^{a',n'} + x_{t-1}^{a',n'}, G_{zk}^{l} z_{t-2}^{GP2,k} + g_k^{l}\omega_{t-2}^{GP2,k} + e_{t-1}^{b,l} + x_{t-1}^{b,l}) \\ &= G_{zk}^{n'} G_{zk}^{l}\sigma_{z^k}^2 + g_k^{n'} g_k^{l}\sigma_{w^k}^2 + \sigma_{e^{n'}e^l} + \sigma_{x^{n'}x^l} \end{aligned}$$

J.2.2 Other covariances

Consanguine relatives (”blood”)

Vertical covariances

Uncle/aunt (siblings of the parents) $aa - b$

We have to compute the covariances between ”aa” and ”b”. Let $n^* = m, f$ be the gender of aa and $l = m, f$ the gender of the b. We project aa on a' (his/her father or mother) who has gender n'

$$Cov(z_t^{aa,n^*}, z_{t-1}^{b,l}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'}, z_{t-1}^{b,l}) = G_{zn'}^{n^*} Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(z_t^{aa,n^*}, y_{t-1}^{b,l}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'}, y_{t-1}^{b,l}) = G_{zn'}^{n^*} Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, z_{t-1}^{b,l}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'}, z_{t-1}^{b,l}) = G_{zn'}^{n^*} Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, y_{t-1}^{b,l}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'}, y_{t-1}^{b,l}) = G_{zn'}^{n^*} Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l})$$

where a' and b are siblings.

Horizontal covariances

Cousins $aa - bb$

We have to compute the covariances between ”aa” and ”bb”. Let $n^* = m, f$ be the gender of aa and $l^* = m, f$ the gender of the bb. We project bb on b (his/her father or mother) who has gender l

$$Cov(z_t^{aa,n^*}, z_t^{bb,l^*}) = Cov(z_t^{aa,n^*}, G_{zl}^{l^*} z_{t-1}^{b,l}) = G_{zl}^{l^*} Cov(z_t^{aa,n^*}, z_{t-1}^{b,l})$$

$$Cov(z_t^{aa,n^*}, y_t^{bb,l^*}) = Cov(z_t^{aa,n^*}, G_{zl}^{l^*} z_{t-1}^{b,l}) = G_{zl}^{l^*} Cov(z_t^{aa,n^*}, z_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, z_t^{bb,l^*}) = Cov(y_t^{aa,n^*}, G_{zl}^{l^*} z_{t-1}^{b,l}) = G_{zl}^{l^*} Cov(y_t^{aa,n^*}, z_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, y_t^{bb,l^*}) = Cov(y_t^{aa,n^*}, G_{zl}^{l^*} z_{t-1}^{b,l}) = G_{zl}^{l^*} Cov(y_t^{aa,n^*}, z_{t-1}^{b,l})$$

where b is the uncle/aunt of aa.

Affinity relatives (”in-law”)

Vertical covariances

Spouse of the uncle/aunt (spouses of the siblings of the parents) $aa - b'$

We have to compute the covariances between ”aa” and ”b'”. Let $n^* = m, f$ be the gender of aa and $l' = m, f$ the gender of the b'. We project b' on his/her spouse b with gender l

$$Cov(z_t^{aa,n^*}, z_{t-1}^{b',l'}) = Cov(z_t^{aa,n^*}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}} \rho_{z^m z^f} z_{t-1}^{b,l}) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}} \rho_{z^m z^f} Cov(z_t^{aa,n^*}, z_{t-1}^{b,l})$$

$$Cov(z_t^{aa,n^*}, y_{t-1}^{b',l'}) = Cov(z_t^{aa,n^*}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(z_t^{aa,n^*}, z_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, z_{t-1}^{b',l'}) = Cov(y_t^{aa,n^*}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(y_t^{aa,n^*}, z_{t-1}^{b,l})$$

$$Cov(y_t^{aa,n^*}, y_{t-1}^{b',l'}) = Cov(y_t^{aa,n^*}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(y_t^{aa,n^*}, z_{t-1}^{b,l})$$

where b is uncle/aunt of aa.

Horizontal covariances

Siblings in law $a - b$

We have to compute the covariances between a and b. Let $n = m, f$ be the gender of a and $l = m, f$ the gender of the b. We project a on his/her spouse a' with gender n'

$$Cov(z_{t-1}^{a,n}, z_{t-1}^{b,l}) = Cov\left(\frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f} z_{t-1}^{a',n'}, z_{t-1}^{b,l}\right) = \frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f} Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(z_{t-1}^{a,n}, y_{t-1}^{b,l}) = Cov\left(\frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f} z_{t-1}^{a',n'}, y_{t-1}^{b,l}\right) = \frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f} Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l})$$

$$Cov(y_{t-1}^{a,n}, z_{t-1}^{b,l}) = Cov\left(\frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f} z_{t-1}^{a',n'}, z_{t-1}^{b,l}\right) = \frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f} Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(y_{t-1}^{a,n'}, y_{t-1}^{b,l}) = Cov\left(\frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f} z_{t-1}^{a',n'}, y_{t-1}^{b,l}\right) = \frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f} Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l})$$

where a' and b are siblings.

We now compute the covariances between "a'" and "b'". Let $n' = m, f$ be the gender of "a'" and $l' = m, f$ the gender of the b'. We project b' on his/her spouse b with gender l

$$Cov(z_{t-1}^{a',n'}, z_{t-1}^{b',l'}) = Cov\left(z_{t-1}^{a',n'}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}\right) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(z_{t-1}^{a',n'}, y_{t-1}^{b',l'}) = Cov\left(z_{t-1}^{a',n'}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}\right) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(y_{t-1}^{a',n'}, z_{t-1}^{b',l'}) = Cov\left(y_{t-1}^{a',n'}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}\right) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(y_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

$$Cov(y_{t-1}^{a',n'}, y_{t-1}^{b',l'}) = Cov\left(y_{t-1}^{a',n'}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}\right) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(y_{t-1}^{a',n'}, z_{t-1}^{b,l})$$

where "a'" and "b" are siblings. Notice that since "a'" is the spouse of "a", $n = f$ when $n' = m$ and viceversa.

Spouse of the siblings in law $a - b'$

We have to compute the covariances between "a" and "b'". Let $n = m, f$ be the gender of a and $l' = m, f$ the gender of the b'. We project b' on his/her spouse b with gender l

$$Cov(z_{t-1}^{a,n}, z_{t-1}^{b',l'}) = Cov(z_{t-1}^{a,n}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(z_{t-1}^{a,n}, z_{t-1}^{b,l})$$

$$Cov(z_{t-1}^{a,n}, y_{t-1}^{b',l'}) = Cov(z_{t-1}^{a,n}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(z_{t-1}^{a,n}, z_{t-1}^{b,l})$$

$$Cov(y_{t-1}^{a,n}, z_{t-1}^{b',l'}) = Cov(y_{t-1}^{a,n}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(y_{t-1}^{a,n}, z_{t-1}^{b,l})$$

$$Cov(y_{t-1}^{a,n}, y_{t-1}^{b',l'}) = Cov(y_{t-1}^{a,n}, \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} z_{t-1}^{b,l}) = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f} Cov(y_{t-1}^{a,n}, z_{t-1}^{b,l})$$

where a and b are siblings in law.

J.3 Correlations

We now compute the correlations in y for different degrees of kinship using the formulas for the covariances we derive above. We denote the correlations by $\rho_{i,n-j,l}$, where i and j denote the corresponding relatives, and n and l are the genders of i and j respectively

Husband and wife $a - a'$

$$\rho_{a-a'} = \frac{\sigma_{z^f}}{\sigma_{y^f}}\frac{\sigma_{z^m}}{\sigma_{y^m}}\rho_{z^f z^m}$$

Parent–child $aa - a'$

$$\rho_{aa,n^*-a',n'} = G_{zn'}^{n^*}\frac{\sigma_{z^{n'}}}{\sigma_{y^{n^*}}}\frac{\sigma_{z^{n'}}}{\sigma_{y^{n'}}}$$

Siblings $a' - b$

$$\rho_{a',n'-b,l} = G_{zk}^{n'}G_{zk}^{l}\frac{\sigma_{z^k}}{\sigma_{y^{n'}}}\frac{\sigma_{z^k}}{\sigma_{y^l}} + g_k^{n'}g_k^l\frac{\sigma_{w^k}}{\sigma_{y^{n'}}}\frac{\sigma_{w^k}}{\sigma_{y^l}} + \frac{\sigma_{e^{n'}e^l}}{\sigma_{y^{n'}}\sigma_{y^l}} + \frac{\sigma_{x^{n'}x^l}}{\sigma_{y^{n'}}\sigma_{y^l}}$$

Uncle/aunt (siblings of the parents) $aa - b$

$$\rho_{aa,n^*-b,l} = G_{zn'}^{n^*}\frac{\sigma_{y^{n'}}}{\sigma_{y^{n^*}}}\left(G_{zk}^{n'}G_{zk}^{l}\frac{\sigma_{z^k}}{\sigma_{y^{n'}}}\frac{\sigma_{z^k}}{\sigma_{y^l}} + g_k^{n'}g_k^l\frac{\sigma_{w^k}}{\sigma_{y^{n'}}}\frac{\sigma_{w^k}}{\sigma_{y^l}} + \frac{\sigma_{e^{n'}e^l}}{\sigma_{y^{n'}}\sigma_{y^l}}\right) \tag{J.3}$$

Cousins $aa - bb$

$$\rho_{aa,n^*-b,l^*} = G_{zl}^{l^*}\frac{\sigma_{y^l}}{\sigma_{y^{l^*}}}\rho_{aa,n^*-b,l}$$

Spouse of the uncle/aunt (spouses of the siblings of the parents) $aa - b'$

$$\rho_{aa,n^*-b',l'} = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f}\frac{\sigma_{y^l}}{\sigma_{y^{l'}}}\rho_{aa,n^*-b,l} \tag{J.4}$$

Siblings in law $a - b$ or $a' - b'$

$$\rho_{a,n-b,l} = \frac{\sigma_{z^n}}{\sigma_{z^{n'}}}\rho_{z^m z^f}\frac{\sigma_{y^{n'}}}{\sigma_{y^n}}\left(G_{zk}^{n'}G_{zk}^{l}\frac{\sigma_{z^k}}{\sigma_{y^{n'}}}\frac{\sigma_{z^k}}{\sigma_{y^l}} + g_k^{n'}g_k^l\frac{\sigma_{w^k}}{\sigma_{y^{n'}}}\frac{\sigma_{w^k}}{\sigma_{y^l}} + \frac{\sigma_{e^{n'}e^l}}{\sigma_{y^{n'}}\sigma_{y^l}}\right)$$

$$\rho_{a',n'-b',l'} = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}\rho_{z^m z^f}\frac{\sigma_{y^l}}{\sigma_{y^{l'}}}\left(G_{zk}^{n'}G_{zk}^{l}\frac{\sigma_{z^k}}{\sigma_{y^{n'}}}\frac{\sigma_{z^k}}{\sigma_{y^l}} + g_k^{n'}g_k^l\frac{\sigma_{w^k}}{\sigma_{y^{n'}}}\frac{\sigma_{w^k}}{\sigma_{y^l}} + \frac{\sigma_{e^{n'}e^l}}{\sigma_{y^{n'}}\sigma_{y^l}}\right) \tag{J.5}$$

Spouse of the siblings in law $a - b'$

$$\rho_{a,n-b',l'} = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}} \rho_{z^m z^f} \frac{\sigma_{y^l}}{\sigma_{y^{l'}}} \rho_{a,n-b,l}$$

J.4 Identification

Since σ_{y^m} and σ_{y^f} are observable, from the ratio of the correlation with the spouses of uncle/aunt to the correlation with the siblings in law, we can identify all the $G_{zn'}^{n^*}$, $n', n^* = m, f$, using different gender combinations. We first substitute the correlation with the uncle/aunt given in (J.3) into (J.4) and we get

$$\rho_{aa,n^*-b',l'} = \frac{\sigma_{z^{l'}}}{\sigma_{z^l}} \rho_{z^m z^f} \frac{\sigma_{y^l}}{\sigma_{y^{l'}}} G_{zn'}^{n^*} \frac{\sigma_{y^{n'}}}{\sigma_{y^{n^*}}} \left(G_{zk}^{n'} G_{zk}^{l} \frac{\sigma_{z^k}}{\sigma_{y^{n'}}} \frac{\sigma_{z^k}}{\sigma_{y^l}} + g_k^{n'} g_k^{l} \frac{\sigma_{w^k}}{\sigma_{y^{n'}}} \frac{\sigma_{w^k}}{\sigma_{y^l}} + \frac{\sigma_{e^{n'} e^l}}{\sigma_{y^{n'}} \sigma_{y^l}} \right)$$

Then we divide this expression by the correlation with the siblings in law given in (J.5) and we get

$$G_{zn'}^{n^*} = \frac{\rho_{aa,n^*-b',l'}}{\rho_{a',n'-b',l'}} \frac{\sigma_{y^{n^*}}}{\sigma_{y^{n'}}}$$

Notice that we have overidentification, since each of the $G_{zn'}^{n^*}$ can be identified using the spouse of the brother of a' or the spouse of the sister of a'.

Next, from the ratio of the uncle in law to the uncle (or the ratio of the spouse of the sibling in law to the sibling in law) we can identify $\rho_{z^m z^f} \frac{\sigma_{z^{l'}}}{\sigma_{z^l}}$

$$\rho_{z^m z^f} \frac{\sigma_{z^{l'}}}{\sigma_{z^l}} = \frac{\rho_{aa,n^*-b',l'}}{\rho_{aa,n^*-b,l}} \frac{\sigma_{y^{l'}}}{\sigma_{y^l}} = \frac{\rho_{a,n-b',l'}}{\rho_{a,n-b,l}} \frac{\sigma_{y^{l'}}}{\sigma_{y^l}}$$

and using the expression above for $l = m$ (then $l' = f$) and $l = f$ (then $l' = m$)

$$\frac{\frac{\rho_{aa,n^*-b',f}}{\rho_{aa,n^*-b,m}} \frac{\sigma_{y^f}}{\sigma_{y^m}}}{\frac{\rho_{aa,n^*-b',m}}{\rho_{aa,n^*-b,f}} \frac{\sigma_{y^m}}{\sigma_{y^f}}} = \frac{\frac{\sigma_{z^f}}{\sigma_{z^m}}}{\frac{\sigma_{z^m}}{\sigma_{z^f}}} = \frac{\sigma_{z^f}^2}{\sigma_{z^m}^2}$$

we can identify $\frac{\sigma_{z^f}^2}{\sigma_{z^m}^2}$ and therefore $\rho_{z^m z^f}$.

Then, we are going to show that using the definition of $G_{zn'}^{n^*}$ for different gender combinations

$$\begin{aligned}
G_{zm}^{m} &= \gamma^m (\alpha_z^m + (1-\alpha_z^m) \frac{\sigma_{z^f}}{\sigma_{z^m}} \rho_{z^m z^f}) \\
G_{zm}^{f} &= \gamma^f (\alpha_z^f + (1-\alpha_z^f) \frac{\sigma_{z^f}}{\sigma_{z^m}} \rho_{z^m z^f}) \\
G_{zf}^{m} &= \gamma^m (\alpha_z^m \frac{\sigma_{z^m}}{\sigma_{z^f}} \rho_{z^m z^f} + (1-\alpha_z^m)) \\
G_{zf}^{f} &= \gamma^f (\alpha_z^f \frac{\sigma_{z^m}}{\sigma_{z^f}} \rho_{z^m z^f} + (1-\alpha_z^f))
\end{aligned}$$

we can identify γ^k and α_z^k, $k = m, f$.

From the expressions for G^m_{zm} and G^m_{zf}

$$\begin{aligned} G^m_{zm} &= \gamma^m(\alpha^m_z + (1-\alpha^m_z)\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f}) = \gamma^m\alpha^m_z + \gamma^m(1-\alpha^m_z)\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f} \\ &= \gamma^m\alpha^m_z\left(1-\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f}\right) + \gamma^m\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f} \end{aligned} \tag{J.6}$$

$$\begin{aligned} G^m_{zf} &= \gamma^m(\alpha^m_z\frac{\sigma_{zm}}{\sigma_{zf}}\rho_{z^m z^f} + (1-\alpha^m_z)) = \gamma^m\alpha^m_z\frac{\sigma_{zm}}{\sigma_{zf}}\rho_{z^m z^f} + \gamma^m(1-\alpha^m_z) \\ &= \gamma^m\alpha^m_z\left(\frac{\sigma_{zm}}{\sigma_{zf}}\rho_{z^m z^f} - 1\right) + \gamma^m \end{aligned} \tag{J.7}$$

Multiplying (J.7) by $\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f}$

$$\begin{aligned} G^m_{zf}\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f} &= \gamma^m\alpha^m_z\left(\frac{\sigma_{zm}}{\sigma_{zf}}\rho_{z^m z^f} - 1\right)\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f} + \gamma^m\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f} \\ &= \gamma^m\alpha^m_z\left(\rho^2_{z^m z^f} - \frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f}\right) + \gamma^m\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f} \end{aligned} \tag{J.8}$$

and subtracting (J.8) from (J.6)

$$G^m_{zm} - G^m_{zf}\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f} = \gamma^m\alpha^m_z(1-\rho^2_{z^m z^f})$$

and therefore

$$\gamma^m\alpha^m_z = \frac{G^m_{zm} - G^m_{zf}\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f}}{1-\rho^2_{z^m z^f}}$$

and

$$\gamma^m = G^m_{zf} - \gamma^m\alpha^m_z\left(\frac{\sigma_{zm}}{\sigma_{zf}}\rho_{z^m z^f} - 1\right)$$

Analogously

$$\gamma^f\alpha^f_z = \frac{G^f_{zm} - G^f_{zf}\frac{\sigma_{zf}}{\sigma_{zm}}\rho_{z^m z^f}}{1-\rho^2_{z^m z^f}}$$

and

$$\gamma^f = G^f_{zf} - \gamma^f\alpha^f_z\left(\frac{\sigma_{zm}}{\sigma_{zf}}\rho_{z^m z^f} - 1\right)$$

Next, from the parent child correlations

$$\frac{\sigma^2_{z^{n'}}}{\sigma^2_{y^{n'}}} = \frac{\rho_{aa,n'-a',n'}}{G^{n'}_{zn'}}$$

we can identify $\frac{\sigma^2_{z^{n'}}}{\sigma^2_{y^{n'}}}$, $n' = m, f$.

Finally, using the eight uncles correlations for different gender combinations $(n^*, n', l = m, f)$

$$\rho_{aa,n^*-b,l} = G^{n^*}_{zn'}\frac{\sigma_{y^{n'}}}{\sigma_{y^{n^*}}}\left(G^{n'}_{zk}G^l_{zk}\frac{\sigma_{z^k}}{\sigma_{y^{n'}}}\frac{\sigma_{z^k}}{\sigma_{y^l}} + g^{n'}_k g^l_k\frac{\sigma_{w^k}}{\sigma_{y^{n'}}}\frac{\sigma_{w^k}}{\sigma_{y^l}} + \frac{\sigma_{e^{n'}e^l}}{\sigma_{y^{n'}}\sigma_{y^l}}\right)$$

we can identify $\sigma^2_{w^k}$, $k = m, f$, $\sigma^2_{e^{n'}}$, $n' = m, f$ and $\sigma_{e^m e^f}$. Notice that again we have overidentification.

Finally from the three siblings correlations for different gender combinations

$$\rho_{a',n'-b,l} = G^{n'}_{zk} G^{l}_{zk} \frac{\sigma_{z^k}}{\sigma_{y^{n'}}} \frac{\sigma_{z^k}}{\sigma_{y^l}} + g^{n'}_k g^l_k \frac{\sigma_{w^k}}{\sigma_{y^{n'}}} \frac{\sigma_{w^k}}{\sigma_{y^l}} + \frac{\sigma_{e^{n'} e^l}}{\sigma_{y^{n'}} \sigma_{y^l}} + \frac{\sigma_{x^{n'} x^l}}{\sigma_{y^{n'}} \sigma_{y^l}}$$

we can identify $\sigma^2_{x^{n'}}$, $n' = m, f$ and $\sigma_{x^m x^f}$.

K The Reduced Form Model

We now study when our general model can be written as a reduced form model where the transmission takes place only through the male line. We consider the following reduced form model where the outcome y for an individual from generation t only depends on his father, and is given by

$$y_t = \beta y^m_{t-1} + z_t + x_t + u_t \tag{K.1}$$

and the socioeconomic status of the child, z_t, only depends on the father z^m_{t-1}

$$z_t = \gamma z^m_{t-1} + e_t + v_t \tag{K.2}$$

Substituting (K.2) in (K.1)

$$y_t = \beta y^m_{t-1} + \gamma z^m_{t-1} + e_t + v_t + x_t + u_t \tag{K.3}$$

Regarding the shocks, as in the general model, we assume that x_t and e_t are shared by all siblings and are uncorrelated to each other and with the other variables (in particular with z^m_{t-1} and y^m_{t-1}). Finally u_t and v_t are white-noise errors.

We can now compare (K.2) and (K.3) with the expression for z^k_t and y^k_t as a function of the father obtained for the general model

$$z^k_t = G^k_{zm} z^m_{t-1} + G^k_{ym} y^m_{t-1} + g^k_m \omega^m_{t-1} + e^k_t + v^k_t \tag{K.4}$$

$$y^k_t = B^k_{ym} y^m_{t-1} + B^k_{zm} z^m_{t-1} + b^k_m \varepsilon^m_{t-1} + g^k_m \omega^m_{t-1} + e^k_t + v^k_t + x^k_t + u^k_t \tag{K.5}$$

where

$$\begin{aligned} G^k_{zm} &= \gamma^k(\alpha^k_z + (1-\alpha^k_z) r^m_{zz}) \\ G^k_{ym} &= \gamma^k(1-\alpha^k_z) r^m_{zy} \\ g^k_m &= \gamma^k(1-\alpha^k_z) \end{aligned}$$

$$B^k_{ym} = \beta^k \left(\alpha^k_y + (1-\alpha^k_y) r^m_{yy}\right) + G^k_{ym}$$

$$
\begin{aligned}
B^k_{zm} &= \beta^k(1-\alpha^k_y)r^m_{yz} + G^k_{zm} \\
b^k_m &= \beta^k(1-\alpha^k_y)
\end{aligned}
$$

It is then easy to see that there are two key differences between the two models:

1. The errors in (K.2) are assumed to be orthogonal to y^m_{t-1} and z^m_{t-1}, whereas in (K.4), z^k_t depends on y^m_{t-1} and therefore, unless $G^k_{ym} = 0$, if we project y^m_t on z^m_{t-1}, the new error will be correlated to y^m_{t-1}.
2. In the reduced form model, z^m_{t-1} has the same coefficient in (K.2) and (K.1), whereas in the general model, the coefficient of z^m_{t-1} is different in (K.4) and (K.5).

Then, we have that the general model can be written as a reduced form model if and only if the following two conditions are satisfied

1. $G^k_{ym} = 0 \Longleftrightarrow \gamma^k = 0$, or $\alpha^k_z = 1$ or $r^m_{zy} = 0$
2. $B^k_{zm} = G^k_{zm} \Longleftrightarrow \beta^k = 0$, or $\alpha^k_y = 1$ or $r^m_{yz} = 0$

We then have that the general model can be written as a reduced form model

1. In the trivial case when just the father matters ($\alpha^k_z = \alpha^k_y = 1$, or $\alpha^k_z = 1$ and $\beta^k = 0$, or $\alpha^k_y = 1$ and $\gamma^k = 0$).
2. When $\beta^k = 0$ and y^m_{t-1} does not influence z^f_{t-1} once the effect of z^m_{t-1} has been netted out ($r^m_{zy} = 0$).
3. When $\gamma^k = 0$ and z^m_{t-1} does not influence y^f_{t-1} once the effect of y^m_{t-1} has been netted out ($r^m_{yz} = 0$).
4. When y^m_{t-1} does not influence z^f_{t-1} once the effect of z^m_{t-1} has been netted out ($r^m_{zy} = 0$), and z^m_{t-1} does not influence y^f_{t-1} once the effect of y^m_{t-1} has been netted out ($r^m_{yz} = 0$).

Notice that Case 2 corresponds to a latent factor model with assortative mating only in z (this is the model we consider in Section J) and Case 3 to a direct effect model with assortative mating only in y. In our baseline specification, our calibrated parameters are close to satisfy the restrictions of Case 2, however, it is worth mentioning than using only male moments we cannot identify all the parameters. From Subsection J.4, we know we can identify G^m_{zm} from the ratio of the correlation with the spouse of the aunt to the correlation with the brother in law

$$
G^m_{zm} = \frac{\rho_{aa,m-b',m}}{\rho_{a',m-b',m}}
$$

and using G^m_{zm} and the father son correlation, we can identify $\frac{\sigma^2_{z^m}}{\sigma^2_{y^{m'}}}$ from

$$
\frac{\sigma^2_{z^m}}{\sigma^2_{y^m}} = \frac{\rho_{aa,m-a',m}}{G^m_{zm}}
$$

However, as it is shown in Subsection J.4, to identify $\rho_{z^m z^f}$ we need the ratio of the correlation with the spouse of the aunt to the correlation with the aunt, and the ratio of the correlation with the spouse of the uncle to the correlation with the uncle, and therefore we need moments involving females. Moreover, to identify γ^m and α_z^m we need to use $G_{zn'}^{n^*}$ for different gender combinations, that is we again need to use moments involving females. Finally to identify $\sigma_{e^m}^2$, we need the correlations between nephew and uncle (brother of the father and brother of the mother), G_{zm}^m and G_{zf}^m and therefore we need female moments to identify G_{zf}^m. Since we cannot identify $\sigma_{e^m}^2$ using only male moments, we cannot identify $\sigma_{x^m}^2$ from the brothers correlation.

L The Genetic Model

The genetic model is nested in our general model by imposing the following restrictions:

- There is no a direct effect of parents outcome on children outcome ($\beta^k = 0$, $k = f, m$), and hence

$$y_t^k = z_t^k + x_t^k + u_t^k$$

  Then $Cov(y_t^k, z_t^k) = \sigma_z^2$ and $\rho_{z^k y^k} = \sigma_z / \sigma_{y^k}$.

- The latent factor is genetic and therefore it is transmitted from parents to children as

$$z_t^k = \frac{z_{t-1}^m + z_{t-1}^f}{2} + v_t^k$$

  where v_t^k is uncorrelated across relatives and to z_{t-1}^m and z_{t-1}^f ($\gamma^k = 1$ and $\sigma_{e_k}^2 = 0$, $k = f, m$).

- The share of the variance explained by the latent factor is equal across genders ($\sigma_{z^k}^2 = \sigma_z^2$, $k = f, m$)

- There is assortative mating only in the observed outcome y ($\rho_{z^m y^f}$, $\rho_{y^m z^f}$ and $\rho_{z^m z^f}$ are functions of $\rho_{y^m y^f}$ and some of the other parameters of the model).

The genetic model has only 5 parameters: $\sigma_z^2, \sigma_{x^m}^2, \sigma_{x^f}^2, \sigma_{x^m x^f}, \rho_{y^m y^f}$.

L.1 Assortative mating process

Under the assumption of assortative mating only in y, the coefficients of the linear projections z_{t-1}^f and y_{t-1}^f on z_{t-1}^m and y_{t-1}^m are

$$r_{zz}^m = \frac{1}{(1-\rho_{z^m y^m}^2)} \frac{\sigma_{z^f}}{\sigma_{z^m}} (\rho_{z^m z^f} - \rho_{z^m y^m} \rho_{y^m z^f}) = 0 \Rightarrow \rho_{z^m z^f} = \rho_{z^m y^m} \rho_{y^m z^f} = \frac{\sigma_z}{\sigma_{y^m}} \rho_{y^m z^f}$$

$$r_{zy}^m = \frac{\sigma_z}{\sigma_{y^m}} \rho_{y^m z^f}$$

$$r_{yz}^m = \frac{1}{(1-\rho_{z^m y^m}^2)} \frac{\sigma_{y^f}}{\sigma_{z^m}} (\rho_{z^m y^f} - \rho_{z^m y^m} \rho_{y^m y^f}) = 0 \Rightarrow \rho_{z^m y^f} = \rho_{z^m y^m} \rho_{y^m y^f} = \frac{\sigma_z}{\sigma_{y^m}} \rho_{y^m y^f}$$

$$r_{yy}^m = \frac{\sigma_{y^f}}{\sigma_{y^m}} \rho_{y^m y^f}$$

and the coefficients of the linear projections of z^m_{t-1} and y^m_{t-1} on z^f_{t-1} and y^f_{t-1} are:

$$r^f_{zz} = \frac{1}{(1-\rho^2_{z^f y^f})}\frac{\sigma_{z^m}}{\sigma_{z^f}}(\rho_{z^m z^f} - \rho_{z^f y^f}\rho_{z^m y^f}) = 0 \Rightarrow \rho_{z^m z^f} = \rho_{z^f y^f}\rho_{z^m y^f} = \frac{\sigma_z}{\sigma_{y^f}}\rho_{z^m y^f} = \frac{\sigma^2_z}{\sigma_{y^f}\sigma_{y^m}}\rho_{y^m y^f}$$

$$r^f_{zy} = \frac{\sigma_z}{\sigma_{y^f}}\rho_{z^m y^f}$$

$$r^f_{yz} = \frac{1}{(1-\rho^2_{z^f y^f})}\frac{\sigma_{y^m}}{\sigma_{z^f}}(\rho_{y^m z^f} - \rho_{z^f y^f}\rho_{y^m y^f}) = 0 \Rightarrow \rho_{y^m z^f} = \rho_{z^f y^f}\rho_{y^m y^f} = \frac{\sigma_z}{\sigma_{y^f}}\rho_{y^m y^f}$$

$$r^f_{yy} = \frac{\sigma_{y^m}}{\sigma_{y^f}}\rho_{y^m y^f}$$

Then, we have that

$$r^m_{zy} = r^f_{zy} = \frac{\sigma_z}{\sigma_{y^m}}\rho_{y^m z^f} = \frac{\sigma^2_z}{\sigma_{y^f}\sigma_{y^m}}\rho_{y^m y^f}$$

$$r^m_{yy} = \frac{\sigma_{y^f}}{\sigma_{y^m}}\rho_{y^m y^f}$$

$$r^f_{zy} = \frac{\sigma_z}{\sigma_{y^f}}\rho_{z^m y^f}$$

and

$$\sigma^2_{w^m} = \sigma^2_z - \left(\frac{\sigma^2_z}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f}\right)^2 \sigma^2_{y^m} = \sigma^2_z\left(1 - \frac{\sigma^2_z}{\sigma^2_{y^f}}\rho^2_{y^m y^f}\right)$$

We can write z^k_t and y^k_t as a function of the father

$$z^k_t = G^k_{zm} z^m_{t-1} + G^k_{ym} y^m_{t-1} + g^k_m \omega^m_{t-1} + v^k_t$$

where

$$G^k_{zm} = \frac{1}{2}\ ,\ G^k_{ym} = \frac{1}{2}\frac{\sigma^2_z}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f}\ ,\ g^k_m = \frac{1}{2}$$

and

$$y^k_t = G^k_{ym} y^m_{t-1} + G^k_{zm} z^m_{t-1} + g^k_m \omega^m_{t-1} + v^k_t + x^k_t + u^k_t$$

and we can write z^k_t and y^k_t as a function of the mother

$$z^k_t = G^k_{zf} z^f_{t-1} + G^k_{yf} y^f_{t-1} + g^k_f \omega^f_{t-1} + v^k_t$$

where

$$G^k_{zf} = \frac{1}{2}\ ,\ G^k_{yf} = \frac{1}{2}\frac{\sigma^2_z}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f}\ ,\ g^k_f = \frac{1}{2}$$

and

$$y^k_t = G^k_{yf} y^f_{t-1} + G^k_{zf} z^f_{t-1} + g^k_f \omega^f_{t-1} + v^k_t + x^k_t + u^k_t$$

L.2 Covariances

L.2.1 Main covariances

We use the notation in Figure 1 to denote individuals with different degrees of kinship. We first compute the main covariances (husband-wife, parent-child and siblings). Then, the covariances for other relatives are obtained recursively.

Husband and wife $a - a'$

We have to compute the covariance between "a" and "a'". Let $n' = m, f$ be the gender of a' and $n = m, f$ the gender of the a.

$$Cov(y_{t-1}^{a,n}, y_{t-1}^{a',n'}) = \sigma_{y^m}\sigma_{y^f}\rho_{y^m y^f}$$

Parent–child $aa - a'$

We have to compute the covariance between "aa" and "a'". Let $n' = m, f$ be the gender of a' and $n^* = m, f$ the gender of the aa. We project aa on a' (his/her father or mother) who has gender n' and we denote by n the gender of the spouse of a'

$$\begin{aligned}
Cov(z_t^{aa,n^*}, z_{t-1}^{a',n'}) &= \frac{1}{2}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)\sigma_z^2 \\
Cov(z_t^{aa,n^*} y_{t-1}^{a',n'}) &= \frac{1}{2}\left(\frac{\sigma_{y^{n'}}}{\sigma_{y^n}}\rho_{y^m y^f} + 1\right)\sigma_{z^{n'}}^2 \\
Cov(y_t^{aa,n^*}, z_{t-1}^{a',n'}) &= \frac{1}{2}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)\sigma_z^2 \\
Cov(y_t^{aa,n^*}, y_{t-1}^{a',n'}) &= \frac{1}{2}\left(\frac{\sigma_{y^{n'}}}{\sigma_{y^n}}\rho_{y^m y^f} + 1\right)\sigma_{z^{n'}}^2
\end{aligned}$$

Siblings $a' - b$

We have to compute the covariance between "a'" and "b". Let $n' = m, f$ be the gender of a' and $l = m, f$ the gender of the b. We project a' and b on their father (or mother) $GP2$ who has gender k, and we denote by k' the gender of the mother (or the father)

$$\begin{aligned}
Cov(z_{t-1}^{a',n'}, z_{t-1}^{b,l}) &= Cov(G_{yk}^{n'} y_{t-2}^{GP2,k} + G_{zk}^{n'} z_{t-2}^{GP2,k} + g_k^{n'}\omega_{t-2}^{GP2,k}, G_{yk}^{l} y_{t-2}^{GP2,k} + G_{zk}^{l} z_{t-2}^{GP2,k} + g_k^{l}\omega_{t-2}^{GP2,k}) \\
&= \left(\frac{1}{2}\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f}\right)^2 \sigma_{y^k}^2 + \frac{1}{4}\sigma_z^2 + \frac{1}{2}\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + \frac{1}{4}\sigma_z^2\left(1 - \frac{\sigma_z^2}{\sigma_{y^{k'}}^2}\rho_{y^m y^f}^2\right) \\
&= \frac{1}{2}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)\sigma_z^2 \\
Cov(z_{t-1}^{a',n'}, y_{t-1}^{b,l}) &= Cov(G_{yk}^{n'} y_{t-2}^{GP2,k} + G_{zk}^{n'} z_{t-2}^{GP2,k} + g_k^{n'}\omega_{t-2}^{GP2,k}, G_{yk}^{l} y_{t-2}^{GP2,k} + G_{zk}^{l} z_{t-2}^{GP2,k} + g_k^{l}\omega_{t-2}^{GP2,k}) \\
&= \frac{1}{2}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)\sigma_z^2 \\
Cov(y_{t-1}^{a',n'}, z_{t-1}^{b,l}) &= Cov(G_{yk}^{n'} y_{t-2}^{GP2,k} + G_{zk}^{n'} z_{t-2}^{GP2,k} + g_k^{n'}\omega_{t-2}^{GP2,k}, G_{yk}^{l} y_{t-2}^{GP2,k} + G_{zk}^{l} z_{t-2}^{GP2,k} + g_k^{l}\omega_{t-2}^{GP2,k}) \\
&= \frac{1}{2}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)\sigma_z^2
\end{aligned}$$

$$
\begin{aligned}
Cov(y_{t-1}^{a',n'}, y_{t-1}^{b,l}) &= Cov(G_{yk}^{n'} y_{t-2}^{GP2,k} + G_{zk}^{n'} z_{t-2}^{GP2,k} + g_k^{n'} \omega_{t-2}^{GP2,k} + x_{t-1}^{n'}, G_{yk}^{l} y_{t-2}^{GP2,k} + G_{zk}^{l} z_{t-2}^{GP2,k} + g_k^{l} \omega_{t-2}^{GP2,k} + x_{t-1}^{l}) \\
&= \frac{1}{2}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)\sigma_z^2 + \sigma_{x^m x^f}
\end{aligned}
$$

L.2.2 Other covariances

Vertical covariances

Uncle/aunt (siblings of the parents) $aa - b$

We have to compute the covariances between "aa" and "b". Let $n^* = m, f$ be the gender of aa and $l = m, f$ the gender of the b. We project aa on a' (his/her father or mother) who has gender n'

$$Cov(z_t^{aa,n^*}, z_{t-1}^{b,l}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'} + G_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{b,l}) = \frac{1}{4}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)^2 \sigma_z^2$$

$$Cov(z_t^{aa,n^*}, y_{t-1}^{b,l}) = Cov(G_{zn'}^{n^*} z_{t-1}^{a',n'} + G_{yn'}^{n^*} y_{t-1}^{a',n'}, y_{t-1}^{b,l}) = \frac{1}{4}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)^2 \sigma_z^2$$

$$Cov(y_t^{aa,n^*}, z_{t-1}^{b,l}) = Cov(B_{zn}^{n^*} z_{t-1}^{a',n'} + B_{yn}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{b,l}) = \frac{1}{4}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)^2 \sigma_z^2$$

$$Cov(y_t^{aa,n^*}, y_{t-1}^{b,l}) = Cov(B_{zn}^{n^*} z_{t-1}^{a',n'} + B_{yn}^{n^*} y_{t-1}^{a',n'}, y_{t-1}^{b,l}) = \frac{1}{4}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)^2 \sigma_z^2 + \frac{1}{2}\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f}\sigma_{x^m x^f}$$

where a' and b are siblings.

Horizontal covariances

Cousins $aa - bb$

We have to compute the covariances between "aa" and "bb". Let $n^* = m, f$ be the gender of aa and $l^* = m, f$ the gender of the ay. We project bb on b (his/her father or mother) who has gender l

$$Cov(z_t^{aa,n^*}, z_t^{bb,l^*}) = Cov(z_t^{aa,n^*}, G_{zl}^{l^*} z_{t-1}^{b,l} + G_{yl}^{l^*} y_{t-1}^{b,l}) = \frac{1}{8}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)^3 \sigma_z^2$$

$$Cov(z_t^{aa,n^*}, y_t^{bb,l^*}) = Cov(z_t^{aa,n^*}, B_{zl}^{l^*} z_{t-1}^{b,l} + B_{yl}^{l^*} y_{t-1}^{b,l}) = \frac{1}{8}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)^3 \sigma_z^2$$

$$Cov(y_t^{aa,n^*}, z_t^{bb,l^*}) = Cov(y_t^{aa,n^*}, G_{zl}^{l^*} z_{t-1}^{b,l} + G_{yl}^{l^*} y_{t-1}^{b,l}) = \frac{1}{8}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)^3 \sigma_z^2$$

$$Cov(y_t^{aa,n^*}, y_t^{bb,l^*}) = Cov(y_t^{aa,n^*}, B_{zl}^{l^*} z_{t-1}^{b,l} + B_{yl}^{l^*} y_{t-1}^{b,l}) = \frac{1}{8}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f} + 1\right)^3 \sigma_z^2 + \frac{1}{4}\left(\frac{\sigma_z^2}{\sigma_{y^m}\sigma_{y^f}}\rho_{y^m y^f}\right)^2 \sigma_{x^m x^f}$$

where b is the uncle/aunt of aa.

L.3 Genetic transmission

Suppose that each person has only one gene with two alleles, A and B. One of the allele is inherited from the father and the other one from the mother. Let X_A and X_B be the random variables representing the potential values each allele may take and suppose that the outcome of interest Y depends on $Z = X_A + X_B$

$$Y = Z + U$$

where U is mean independent of Z.

We have to compute

$$E\left(Z_t \,|\, Z_{t-1}^m = z^m,\ Z_{t-1}^f = z^f\right)$$

The distribution of the child Z_t conditional on the parents $X_{A,t-1}^m = x_A^m,\ X_{B,t-1}^m = x_B^m,\ X_{A,t-1}^f = x_A^f,$ $X_{B,t-1}^f = x_B^f$ is multinomial with the following probability mass function

$$Z_t = \begin{cases} x_A^m + x_A^f, \text{ with probability } \frac{1}{4} \\ x_A^m + x_B^f, \text{ with probability } \frac{1}{4} \\ x_B^m + x_A^f, \text{ with probability } \frac{1}{4} \\ x_B^m + x_B^f, \text{ with probability } \frac{1}{4} \end{cases}$$

Then, the distribution of Z_t conditional on $X_{A,t-1}^m = x_A^m,\ Z_{t-1}^m = z^m,\ X_{A,t-1}^f = x_A^f,\ Z_{t-1}^f = z^f$ is also multinomial, and the probability mass function is

$$Z_t = \begin{cases} x_A^m + x_A^f, \text{ with probability } \frac{1}{4} \\ x_A^m + z^f - x_A^f, \text{ with probability } \frac{1}{4} \\ z^m - x_A^m + x_A^f, \text{ with probability } \frac{1}{4} \\ z^m - x_A^m + z^f - x_A^f, \text{ with probability } \frac{1}{4} \end{cases}$$

Then,

$$\begin{aligned} E\left(Z_t \,|\, X_{A,t-1}^m,\ Z_{t-1}^m,\ X_{A,t-1}^f,\ Z_{t-1}^f\right) &= \frac{1}{4}\left(X_{A,t-1}^m + X_{A,t-1}^f\right) + \frac{1}{4}\left(X_{A,t-1}^m + Z_{t-1}^f - X_{A,t-1}^f\right) \\ &+ \frac{1}{4}\left(Z_{t-1}^m - X_{A,t-1}^m + X_{A,t-1}^f\right) \frac{1}{4}\left(Z_{t-1}^m - X_{A,t-1}^m + Z_{t-1}^f - X_{A,t-1}^f\right) \\ &= \frac{1}{4} Z_{t-1}^f + \frac{1}{4} Z_{t-1}^m + \frac{1}{4}\left(Z_{t-1}^m + Z_{t-1}^f\right) = \frac{1}{2}\left(Z_{t-1}^m + Z_{t-1}^f\right) \end{aligned}$$

Since $E\left(Z_t \,|\, X_{A,t-1}^m,\ Z_{t-1}^m,\ X_{A,t-1}^f,\ Z_{t-1}^f\right)$ does not depend on $X_{A,t-1}^m$ and $X_{A,t-1}^f$, using the law of iterated expectations

$$E\left(Z_t \,|\, Z_{t-1}^m,\ Z_{t-1}^f\right) = \frac{1}{2}\left(Z_{t-1}^m + Z_{t-1}^f\right)$$

If we now consider that each person has n genes, each with two alleles, A_i and B_i. For each gene, one of the allele is inherited from the father and the other one from the mother. Let X_{A_i} and X_{B_i} be the random variables representing the potential values each allele may take and suppose that the outcome of interest Y depends on $Z = \sum_{i=1}^n (X_{A_i} + X_{B_i}) = \sum_{i=1}^n Z_i$, where $Z_i = X_{A_i} + X_{B_i}$. For each gene i, given $X_{A_i,t-1}^m = x_{A_i}^m,\ Z_{i,t-1}^m = z_i^m,\ X_{A_i,t-1}^f = x_{A_i}^f$, there are 4 possible realizations of the child alleles of gene

i, and therefore 4^n possible genomes. Then, the distribution of the child Z_t conditional on the parents $X^m_{A_1,t-1} = x^m_{A_1}$, $Z^m_{1,t-1} = z^m_1$, $X^f_{A_1,t-1} = x^f_{A_1}$, $Z^f_{1,t-1} = z^f_1$, ...,$X^m_{A_n,t-1} = x^m_{A_n}$, $Z^m_{n,t-1} = z^m_n$, $X^f_{A_n,t-1} = x^f_{A_n}$, $Z^f_{n,t-1} = z^f_n$ is also multinomial, and the probability mass function is

$$Z_t = \begin{cases} \left(x^m_{A_1} + x^f_{A_1}\right) + \left(x^m_{A_2} + x^f_{A_2}\right) + .. + \left(x^m_{A_n} + x^f_{A_n}\right), \text{ with probability } \frac{1}{4^n} \\ \left(x^m_{A_1} + z^f_1 - x^f_{A_1}\right) + \left(x^m_{A_2} + x^f_{A_2}\right) + .. + \left(x^m_{A_n} + x^f_{A_n}\right), \text{ with probability } \frac{1}{4^n} \\ \left(z^m_1 - x^m_{A_1} + x^f_{A_1}\right) + \left(x^m_{A_2} + x^f_{A_2}\right) + .. + \left(x^m_{A_n} + x^f_{A_n}\right), \text{ with probability } \frac{1}{4^n} \\ \left(z^m_1 - x^m_{A_1} + z^f_1 - x^f_{A_1}\right) + \left(x^m_{A_2} + x^f_{A_2}\right) + .. + \left(x^m_{A_n} + x^f_{A_n}\right), \text{ with probability } \frac{1}{4^n} \\ \vdots \\ \left(z^m_1 - x^m_{A_1} + z^f_1 - x^f_{A_1}\right) + \left(z^m_2 - x^m_{A_2} + z^f_2 - x^f_{A_2}\right) + .. + \left(z^m_n - x^m_{A_n} + z^f_n - x^f_{A_n}\right), \text{ with probability } \frac{1}{4^n} \end{cases}$$

As in the model with just one gene, all the $x^k_{A_i}$ cancel when we compute the conditional mean

$$\begin{aligned} & E\left(Z_t \,|\, X^m_{A_1,t-1}, Z^m_{1,t-1}, X^f_{A_1,t-1}, Z^f_{1,t-1}, ..., X^m_{A_n,t-1}, Z^m_{n,t-1}, X^f_{A_n,t-1}, Z^f_{n,t-1}\right) \\ = \; & \frac{1}{2} Z^m_{1,t-1} + ... + \frac{1}{2} Z^m_{n,t-1} + \frac{1}{2} Z^f_{1,t-1} + ... + \frac{1}{2} Z^f_{n,t-1} = \frac{1}{2}\left(Z^m_{t-1} + Z^f_{t-1}\right) \end{aligned}$$

Then, since the conditional expectation above only depends on Z^m_{t-1} and Z^f_{t-1}, using the law of iterated expectations

$$E\left(Z_t \,|\, Z^m_{t-1},\ Z^f_{t-1}\right) = \frac{1}{2}\left(Z^m_{t-1} + Z^f_{t-1}\right)$$

Let us now consider two siblings i and j. We have to compute

$$E\left(Z_{it} Z_{jt} \,|\, Z^m_{t-1} = z^m,\, Z^f_{t-1} = z^f\right)$$

The distribution of $Z_{it}Z_{jt}$ conditional on the parents $X^m_{A,t-1} = x^m_A$, $X^m_{B,t-1} = x^m_B$, $X^f_{A,t-1} = x^f_A$, $X^f_{B,t-1} = x^f_B$ is multinomial with the following probability mass function

$$Z_{it}Z_{jt} = \begin{cases} \left(x^m_A + x^f_A\right)^2, \text{ with probability } \frac{1}{16} \\ \left(x^m_A + x^f_A\right)\left(x^m_A + x^f_B\right), \text{ with probability } \frac{1}{8} \\ \left(x^m_A + x^f_A\right)\left(x^m_B + x^f_A\right), \text{ with probability } \frac{1}{8} \\ \left(x^m_A + x^f_A\right)\left(x^m_B + x^f_B\right), \text{ with probability } \frac{1}{8} \\ \left(x^m_A + x^f_B\right)^2, \text{ with probability } \frac{1}{16} \\ \left(x^m_A + x^f_B\right)\left(x^m_B + x^f_A\right), \text{ with probability } \frac{1}{8} \\ \left(x^m_A + x^f_B\right)\left(x^m_B + x^f_B\right), \text{ with probability } \frac{1}{8} \\ \left(x^m_B + x^f_A\right)^2, \text{ with probability } \frac{1}{16} \\ \left(x^m_B + x^f_A\right)\left(x^m_B + x^f_B\right), \text{ with probability } \frac{1}{8} \\ \left(x^m_B + x^f_B\right)^2, \text{ with probability } \frac{1}{16} \end{cases}$$

Then, the distribution of $Z_{it}Z_{jt}$ conditional on $X^m_{A,t-1} = x^m_A$, $Z^m_{t-1} = z^m$, $X^f_{A,t-1} = x^f_A$, $Z^f_{t-1} = z^f$ is also multinomial, and the probability mass function is

$$Z_{it}Z_{jt} = \begin{cases} \left(x^m_A + x^f_A\right)^2, \text{ with probability } \frac{1}{16} \\ \left(x^m_A + x^f_A\right)\left(x^m_A + z^f - x^f_A\right), \text{ with probability } \frac{1}{8} \\ \left(x^m_A + x^f_A\right)\left(z^m - x^m_A + x^f_A\right), \text{ with probability } \frac{1}{8} \\ \left(x^m_A + x^f_A\right)\left(z^m - x^m_A + z^f - x^f_A\right), \text{ with probability } \frac{1}{8} \\ \left(x^m_A + z^f - x^f_A\right)^2, \text{ with probability } \frac{1}{16} \\ \left(x^m_A + z^f - x^f_A\right)\left(z^m - x^m_A + x^f_A\right), \text{ with probability } \frac{1}{8} \\ \left(x^m_A + z^f - x^f_A\right)\left(z^m - x^m_A + z^f - x^f_A\right), \text{ with probability } \frac{1}{8} \\ \left(z^m - x^m_A + x^f_A\right)^2, \text{ with probability } \frac{1}{16} \\ \left(z^m - x^m_A + x^f_A\right)\left(z^m - x^m_A + z^f - x^f_A\right), \text{ with probability } \frac{1}{8} \\ \left(z^m - x^m_A + z^f - x^f_A\right)^2, \text{ with probability } \frac{1}{16} \end{cases}$$

Then,

$$\begin{aligned} & E\left(Z_{it}Z_{jt} \,|\, X^m_{A,t-1},\ Z^m_{t-1},\ X^f_{A,t-1},\ Z^f_{t-1}\right) \\ = & \frac{1}{16}\left(X^m_A + X^f_A\right)\left(\left(X^m_A + X^f_A\right) + \left(X^m_A + Z^f - X^f_A\right) + \left(Z^m - X^m_A + X^f_A\right) + \left(Z^m - X^m_A + Z^f - X^f_A\right)\right) \\ & + \frac{1}{16}\left(X^m_A + Z^f - X^f_A\right)\left(\left(X^m_A + X^f_A\right) + \left(X^m_A + Z^f - X^f_A\right) + \left(Z^m - X^m_A + X^f_A\right) + \left(Z^m - X^m_A + Z^f - X^f_A\right)\right) \\ & + \frac{1}{16}\left(Z^m - X^m_A + X^f_A\right)\left(\left(X^m_A + X^f_A\right) + \left(X^m_A + Z^f - X^f_A\right) + \left(Z^m - X^m_A + X^f_A\right) + \left(Z^m - X^m_A + Z^f - X^f_A\right)\right) \\ & + \frac{1}{16}\left(Z^m - X^m_A + Z^f - X^f_A\right)\left(\left(X^m_A + X^f_A\right) + \left(X^m_A + Z^f - X^f_A\right) + \left(Z^m - X^m_A + X^f_A\right) + \left(Z^m - X^m_A + Z^f - X^f_A\right)\right) \\ = & \frac{1}{8}\left(X^m_A + X^f_A\right)\left(Z^f + Z^m\right) + \frac{1}{8}\left(X^m_A + Z^f - X^f_A\right)\left(Z^f + Z^m\right) \\ & + \frac{1}{8}\left(Z^m - X^m_A + X^f_A\right)\left(Z^f + Z^m\right) + \frac{1}{8}\left(Z^m - X^m_A + Z^f - X^f_A\right)\left(Z^f + Z^m\right) \\ = & \frac{1}{4}\left(Z^m_{t-1} + Z^f_{t-1}\right)^2 \end{aligned}$$

Since $E\left(Z_{it}Z_{jt} \,|\, X^m_{A,t-1},\ Z^m_{t-1},\ X^f_{A,t-1},\ Z^f_{t-1}\right)$ does not depend on $X^m_{A,t-1}$ and $X^f_{A,t-1}$, using the law of iterated expectations

$$E\left(Z_{it}Z_{jt}|\, Z^m_{t-1},\ Z^f_{t-1}\right) = \frac{1}{4}\left(Z^m_{t-1} + Z^f_{t-1}\right)^2$$

We then have that $E\left(Z_{it}Z_{jt}|\, Z^m_{t-1},\ Z^f_{t-1}\right) = E\left(Z_{it}|\, Z^m_{t-1},\ Z^f_{t-1}\right) E\left(Z_{jt}|\, Z^m_{t-1},\ Z^f_{t-1}\right)$ and Z_{it} and Z_{jt} are uncorrelated conditional on Z^m_{t-1}, Z^f_{t-1}. Then, we can write

$$Z_{it} = \frac{1}{2}\left(Z^m_{t-1} + Z^f_{t-1}\right) + e_{it}$$

where e_{it} and e_{jt} are uncorrelated across siblings.

M The General Model with Two Unobservable Factors

We assume that the value of the output y for an individual from generation t is given by

$$y_t^k = \beta^k \widetilde{y}_{t-1}^k + z_t^{G,k} + z_t^{C,k} + x_t^k + u_t^k \tag{M.1}$$

where the superscript k stands for males ($k = m$) and for females ($k = f$). We assume that

$$\widetilde{y}_{t-1}^k = \alpha_y^k y_{t-1}^m + (1 - \alpha_y^k) y_{t-1}^f$$

$z_t^{G,k}$ and $z_t^{C,k}$ are two unobservable factors. The genetic factor of the child, $z_t^{G,k}$, depends on the father $z_{t-1}^{G,m}$ as well as on the mother $z_{t-1}^{G,f}$

$$z_t^{G,k} = \frac{z_{t-1}^{G,m} + z_{t-1}^{G,f}}{2} + v_t^{G,k} \tag{M.2}$$

The cultural factor of the child, $z_t^{C,k}$, also depends on the father $z_{t-1}^{C,m}$ as well as on the mother $z_{t-1}^{C,f}$

$$\begin{aligned} z_t^{C,k} &= \gamma^k \widetilde{z}_{t-1}^k + e_t^{C,k} + v_t^{C,k} \\ \widetilde{z}_{t-1}^k &= \alpha_z^k z_{t-1}^{C,m} + (1 - \alpha_z^k) z_{t-1}^{C,f} \end{aligned} \tag{M.3}$$

Regarding the shocks in model (M.1), we assume that x_t^k, and $e_t^{C,k}$ are shared by all siblings of the same gender, can be correlated across siblings of different gender and are uncorrelated with the other variables (in particular with $z_t^{G,k}$, $z_t^{C,k}$and y_{t-1}). Finally u_t^k, $v_t^{G,k}$ and $v_t^{C,k}$ are individual's white-noise error terms.

M.1 Assortative mating process

We assume there is assortative mating both in years of schooling and in the cultural factor. In particular we consider the linear projections of $z_{t-1}^{G,f}$, $z_{t-1}^{C,f}$ and y_{t-1}^f on $z_{t-1}^{G,m}$, $z_{t-1}^{C,m}$ and y_{t-1}^m:

$$\begin{aligned} z_{t-1}^{G,f} &= r_{z^G z^G}^m \; z_{t-1}^{G,m} + r_{z^G z^C}^m \; z_{t-1}^{C,m} + r_{z^G y}^m \; y_{t-1}^m + w_{t-1}^{G,m} \\ z_{t-1}^{C,f} &= r_{z^C z^G}^m \; z_{t-1}^{G,m} + r_{z^C z^C}^m \; z_{t-1}^{C,m} + r_{z^C y}^m \; y_{t-1}^m + w_{t-1}^{C,m} \\ y_{t-1}^f &= r_{y z^G}^m \; z_{t-1}^{G,m} + r_{y z^C}^m \; z_{t-1}^{C,m} + r_{yy}^m \; y_{t-1}^m + \varepsilon_{t-1}^m \end{aligned}$$

The coefficients of the linear projections depend on 15 correlations, $\rho_{z^{G,m} z^{C,m}}$, $\rho_{z^{G,m} y^m}$, $\rho_{z^{C,m} y^m}$, $\rho_{z^{G,f} z^{C,f}}$, $\rho_{z^{G,f} y^f}$, $\rho_{z^{C,f} y^f}$, $\rho_{z^{G,m} z^{G,f}}$, $\rho_{z^{G,m} z^{C,f}}$, $\rho_{z^{G,m} y^{,f}}$, $\rho_{z^{C,m} z^{G,f}}$, $\rho_{z^{C,m} z^{C,f}}$, $\rho_{z^{C,m} y^{,f}}$, $\rho_{y^m z^{G,f}}$, $\rho_{y^m z^{C,f}}$ and $\rho_{y^m y^f}$, as well as on the standard deviations of $z_{t-1}^{G,k}$, $z_{t-1}^{C,k}$ and y_{t-1}^k, $k = m, f$. However, since we assume there is no assortative mating in the genetic factor, we have that $r_{z^G z^G}^m = r_{z^C z^G}^m = r_{y z^G}^m = 0$, which implies

$$\rho_{z^{G,m} z^{G,f}} = \frac{\left(\rho_{z^{G,m} y^m} \rho_{z^{C,m} y^m} - \rho_{z^{G,m} z^{C,m}}\right) \rho_{z^{C,m} z^{G,f}} + \left(\rho_{z^{G,m} z^{C,m}} \rho_{z^{C,m} y^m} - \rho_{z^{G,m} y^m}\right) \rho_{y^m z^{G,f}}}{\rho_{z^{C,m} y^m}^2 - 1} \tag{M.4}$$

$$\rho_{z^{G,m}z^{C,f}} = \frac{\left(\rho_{z^{G,m}y^m}\rho_{z^{C,m}y^m} - \rho_{z^{G,m}z^{C,m}}\right)\rho_{z^{C,m}z^{C,f}} + \left(\rho_{z^{G,m}z^{C,m}}\rho_{z^{C,m}y^m} - \rho_{z^{G,m}y^m}\right)\rho_{y^m z^{C,f}}}{\rho^2_{z^{C,m}y^m} - 1}$$

$$\rho_{z^{G,m}y^f} = \frac{\left(\rho_{z^{G,m}y^m}\rho_{z^{C,m}y^m} - \rho_{z^{G,m}z^{C,m}}\right)\rho_{z^{C,m}y^f} + \left(\rho_{z^{G,m}z^{C,m}}\rho_{z^{C,m}y^m} - \rho_{z^{G,m}y^m}\right)\rho_{y^m y^f}}{\rho^2_{z^{C,m}y^m} - 1}$$

which reduces the number of free correlations to 12.

The remaining coefficients are:

$$r^m_{z^G z^C} = \frac{1}{(1-\rho^2_{z^{C,m}y^m})}\frac{\sigma_{z^{G,f}}}{\sigma_{z^{C,m}}}(\rho_{z^{C,m}z^{G,f}} - \rho_{z^{C,m}y^m}\rho_{y^m z^{G,f}})$$

$$r^m_{z^G y} = \frac{1}{(1-\rho^2_{z^{C,m}y^m})}\frac{\sigma_{z^{G,f}}}{\sigma_{y^m}}(\rho_{y^m z^{G,f}} - \rho_{z^{C,m}y^m}\rho_{z^{C,m}z^{G,f}})$$

$$r^m_{z^C z^C} = \frac{1}{(1-\rho^2_{z^{C,m}y^m})}\frac{\sigma_{z^{C,f}}}{\sigma_{z^{C,m}}}(\rho_{z^{C,m}z^{C,f}} - \rho_{z^{C,m}y^m}\rho_{y^m z^{C,f}})$$

$$r^m_{z^C y} = \frac{1}{(1-\rho^2_{z^{C,m}y^m})}\frac{\sigma_{z^{C,f}}}{\sigma_{y^m}}(\rho_{y^m z^{C,f}} - \rho_{z^{C,m}y^m}\rho_{z^{C,m}z^{C,f}})$$

$$r^m_{yz^C} = \frac{1}{(1-\rho^2_{z^{C,m}y^m})}\frac{\sigma_{y^f}}{\sigma_{z^{C,m}}}(\rho_{z^{C,m}y^f} - \rho_{z^{C,m}y^m}\rho_{y^m y^f})$$

$$r^m_{yy} = \frac{1}{(1-\rho^2_{z^{C,m}y^m})}\frac{\sigma_{y^f}}{\sigma_{y^m}}(\rho_{y^m y^f} - \rho_{z^{C,m}y^m}\rho_{z^{C,m}y^f})$$

We use these matching functions to write the genetic factor, $z^{G,k}_t$, the cultural factor, $z^{C,k}_t$, and years of schooling, y^k_t, as a function of father's genetic factor, $z^{G,m}_{t-1}$, cultural factor, $z^{C,m}_{t-1}$, and years of schooling, y^m_{t-1}. We write (M.2) as

$$\begin{aligned} z^{G,k}_t &= \frac{z^{G,m}_{t-1} + z^{G,f}_{t-1}}{2} + v^{G,k}_t \\ &= \frac{1}{2}\left(z^{G,m}_{t-1} + r^m_{z^G z^C} z^{C,m}_{t-1} + r^m_{z^G y} y^m_{t-1} + w^{G,m}_{t-1}\right) + v^{G,k}_t \\ &= G^k_{zgm} z^{G,m}_{t-1} + G^k_{zm} z^{C,m}_{t-1} + G^k_{ym} y^m_{t-1} + g^k_m w^{G,m}_{t-1} + v^{G,k}_t \end{aligned}$$

where

$$\begin{aligned} G^k_{zgm} &= \frac{1}{2} \\ G^k_{zm} &= \frac{1}{2} r^m_{z^G z^C} \\ G^k_{ym} &= \frac{1}{2} r^m_{z^G y} \\ g^k_m &= \frac{1}{2} \end{aligned}$$

(M.3) as

$$z^{C,k}_t = \gamma^k\left(\alpha^k_z z^{C,m}_{t-1} + (1-\alpha^k_z) z^{C,f}_{t-1}\right) + e^{C,k}_t + v^{C,k}_t$$

$$
\begin{aligned}
&= \gamma^k \left(\alpha_z^k z_{t-1}^{C,m} + (1-\alpha_z^k)\left(r^m_{z^C z^C} z_{t-1}^{C,m} + r^m_{z^C y} y^m_{t-1} + w^{C,m}_{t-1}\right)\right) + e_t^{C,k} + v_t^{C,k} \\
&= C^k_{zm} z^{C,m}_{t-1} + C^k_{ym} y^m_{t-1} + c^k_m \omega^{C,m}_{t-1} + e_t^{C,k} + v_t^{C,k}
\end{aligned}
$$

where

$$
\begin{aligned}
C^k_{zm} &= \gamma^k(\alpha_z^k + (1-\alpha_z^k) r^m_{z^C z^C}) \\
C^k_{ym} &= \gamma^k(1-\alpha_z^k) r^m_{z^C y} \\
c^k_m &= \gamma^k(1-\alpha_z^k)
\end{aligned}
$$

and (M.1) as

$$
y_t^k = \beta^k \left(\alpha_y^k y^m_{t-1} + (1-\alpha_y^k) y^f_{t-1}\right) + z_t^{G,k} + z_t^{C,k} + x_t^k + u_t^k
$$

$$
\begin{aligned}
y_t^k &= \beta^k \left(\alpha_y^k y^m_{t-1} + (1-\alpha_y^k) y^f_{t-1}\right) + z_t^{G,k} + z_t^{C,k} + x_t^k + u_t^k \\
&= \beta^k \left(\alpha_y^k y^m_{t-1} + (1-\alpha_y^k)\left(r^m_{yz^C} z^{C,m}_{t-1} + r^m_{yy} y^m_{t-1} + \varepsilon^m_{t-1}\right)\right) \\
&\quad \frac{1}{2} z^{G,m}_{t-1} + \frac{1}{2} r^m_{z^G z^C} z^{C,m}_{t-1} + \frac{1}{2} r^m_{z^G y} y^m_{t-1} + \frac{1}{2} w^{G,m}_{t-1} + v_t^{G,k} \\
&\quad C^k_{zm} z^{C,m}_{t-1} + C^k_{ym} y^m_{t-1} + c^k_m \omega^{C,m}_{t-1} + e_t^{C,k} + v_t^{C,k} + x_t^k + u_t^k
\end{aligned}
$$

$$
\begin{aligned}
y_t^k &= B^k_{zgm} z^{G,m}_{t-1} + B^k_{zm} z^{C,m}_{t-1} + B^k_{ym} y^m_{t-1} + g^k_m w^{G,m}_{t-1} + c^k_m \omega^{C,m}_{t-1} + b^k_m \varepsilon^m_{t-1} \\
&\quad + e_t^{G,k} + e_t^{C,k} + x_t^k + v_t^{G,k} + v_t^{C,k} + u_t^k
\end{aligned}
$$

where

$$
\begin{aligned}
B^k_{zgm} &= \frac{1}{2} \\
B^k_{zm} &= \beta^k(1-\alpha_y^k) r^m_{yz^C} + \frac{1}{2} r^m_{z^G z^C} + C^k_{zm} \\
B^k_{ym} &= \beta^k \left(\alpha_y^k + (1-\alpha_y^k) r^m_{yy}\right) + \frac{1}{2} r^m_{z^G y} + C^k_{ym} \\
b^k_m &= \beta^k(1-\alpha_y^k)
\end{aligned}
$$

All these expressions will be used to compute correlations between relatives that are related through their fathers.

Analogously, we can compute the linear projections of $z^{G,m}_{t-1}$, $z^{C,m}_{t-1}$ and y^m_{t-1} on $z^{G,f}_{t-1}$, $z^{C,f}_{t-1}$ and y^f_{t-1}. The assumption of no assortative mating in the genetic factor implies $r^f_{z^G z^G} = r^f_{z^C z^G} = r^f_{yz^G} = 0$

$$
\rho_{z^{G,m} z^{G,f}} = \frac{\left(\rho_{z^{G,f} y^f} \rho_{z^{C,f} y^f} - \rho_{z^{G,f} z^{C,f}}\right) \rho_{z^{G,m} z^{C,f}} + \left(\rho_{z^{G,f} z^{C,f}} \rho_{z^{C,f} y^f} - \rho_{z^{G,f} y^f}\right) \rho_{z^{G,m} y^f}}{\rho^2_{z^{C,f} y^f} - 1} \tag{M.5}
$$

$$\rho_{z^{C,m}z^{G,f}} = \frac{\left(\rho_{z^{G,f}y^f}\rho_{z^{C,f}y^f} - \rho_{z^{G,f}z^{C,f}}\right)\rho_{z^{C,m}z^{C,f}} + \left(\rho_{z^{G,f}z^{C,f}}\rho_{z^{C,f}y^f} - \rho_{z^{G,f}y^f}\right)\rho_{z^{C,m}y^f}}{\rho^2_{z^{C,f}y^f} - 1}$$

$$\rho_{y^m z^{G,f}} = \frac{\left(\rho_{z^{G,f}y^f}\rho_{z^{C,f}y^f} - \rho_{z^{G,f}z^{C,f}}\right)\rho_{y^m z^{C,f}} + \left(\rho_{z^{G,f}z^{C,f}}\rho_{z^{C,f}y^f} - \rho_{z^{G,f}y^f}\right)\rho_{y^m y^f}}{\rho^2_{z^{C,f}y^f} - 1}$$

The definitions of $\rho_{z^{G,m}z^{G,f}}$ in (M.4) and (M.5) turn to be identical and therefore the the number of free correlations is reduced to 10. We can then write the genetic factor, $z_t^{G,k}$, the cultural factor, $z_t^{C,k}$, and years of schooling, y_t^k, as a function of mother's genetic factor, $z_{t-1}^{G,f}$, cultural factor, $z_{t-1}^{C,f}$, and years of schooling, y_{t-1}^f.These expressions will be used to compute correlations between relatives that are related through their mothers.

M.2 Steady state assumption

We assume that the second order moments of all variables are time invariant. This steady state assumption implies that $\rho_{z^{C,m}z^{G,m}}$, $\rho_{z^{C,m}y^m}$, $\rho_{z^{G,m}y^m}$, $\rho_{z^{C,f}z^{G,f}}$, $\rho_{z^{C,f}y^f}$, and $\rho_{z^{G,f}y^f}$ depend on the remaining parameters of the model as shown below.

We first compute $\rho_{z^{G,m}z^{C,m}}$

$$\begin{aligned}
Cov(z_t^{C,m}, z_t^{G,m}) &= Cov\left(\gamma^m\alpha_z^m z_{t-1}^{C,m} + \gamma^m(1-\alpha_z^m)z_{t-1}^{C,f}, \frac{1}{2}z_{t-1}^{G,m} + \frac{1}{2}z_{t-1}^{G,f}\right) \\
&= \gamma^m\alpha_z^m\frac{1}{2}Cov\left(z_{t-1}^{C,m}, z_{t-1}^{G,m}\right) + \gamma^m\alpha_z^m\frac{1}{2}Cov\left(z_{t-1}^{C,m}, z_{t-1}^{G,f}\right) \\
&\quad + \gamma^m(1-\alpha_z^m)\frac{1}{2}Cov\left(z_{t-1}^{C,f}, z_{t-1}^{G,m}\right) + \gamma^m(1-\alpha_z^m)\frac{1}{2}Cov\left(z_{t-1}^{C,f}, z_{t-1}^{G,f}\right)
\end{aligned}$$

and dividing by $\sigma_{z^{C,m}}$ and $\sigma_{z^{G,m}}$ and rearranging we have

$$\begin{aligned}
&\left(1 - \gamma^m\alpha_z^m\frac{1}{2}\right)\rho_{z^{G,m}z^{C,m}} - \gamma^m(1-\alpha_z^m)\frac{1}{2}\frac{\sigma_{z^{C,f}}}{\sigma_{z^{C,m}}}\frac{\sigma_{z^{G,f}}}{\sigma_{z^{G,m}}}\rho_{z^{C,f}z^{G,f}} \\
= \;& \gamma^m\alpha_z^m\frac{1}{2}\frac{\sigma_{z^{G,f}}}{\sigma_{z^{G,m}}}\rho_{z^{C,m}z^{G,f}} + \gamma^m(1-\alpha_z^m)\frac{1}{2}\frac{\sigma_{z^{C,f}}}{\sigma_{z^{C,m}}}\rho_{z^{G,m}z^{C,f}}
\end{aligned}$$

analogously we obtain another equation from the steady state assumption on $\rho_{z^{G,f}z^{C,f}}$

$$\begin{aligned}
&\left(1 - \frac{1}{2}\gamma^f\left(1-\alpha_z^f\right)\right)\rho_{z^{G,f}z^{C,f}} - \frac{1}{2}\gamma^f\alpha_z^f\frac{\sigma_{z^{C,m}}}{\sigma_{z^{C,f}}}\frac{\sigma_{z^{G,m}}}{\sigma_{z^{G,f}}}\rho_{z^{G,m}z^{C,m}} \\
= \;& \frac{1}{2}\gamma^f\left(1-\alpha_z^f\right)\frac{\sigma_{z^{G,m}}}{\sigma_{z^{G,f}}}\rho_{z^{G,m}z^{C,f}} + \frac{1}{2}\gamma^f\alpha_z^f\frac{\sigma_{z^{C,m}}}{\sigma_{z^{C,f}}}\rho_{z^{C,m}z^{G,f}}
\end{aligned}$$

We now compute $\rho_{z^{G,m}y^m}$

$$\begin{aligned}
Cov(y_t^m, z_t^{G,m}) &= Cov\left(\beta^m\tilde{y}_{t-1}^m + z_t^{G,m} + z_t^{C,m}, z_t^{G,m}\right) = Cov\left(\beta^m\alpha_y^m y_{t-1}^m + \beta^m(1-\alpha_y^m)y_{t-1}^f, \frac{1}{2}z_{t-1}^{G,m} + \frac{1}{2}z_{t-1}^{G,f}\right) \\
&\quad + \sigma^2_{z^{G,m}} + Cov\left(z_t^{C,m}, z_t^{G,m}\right) \\
&= \beta^m\alpha_y^m\frac{1}{2}Cov\left(y_{t-1}^m, z_{t-1}^{G,m}\right) + \beta^m\alpha_y^m\frac{1}{2}Cov\left(y_{t-1}^m, z_{t-1}^{G,f}\right) + \beta^m(1-\alpha_y^m)\frac{1}{2}Cov\left(y_{t-1}^f, z_{t-1}^{G,m}\right)
\end{aligned}$$

$$+\beta^m(1-\alpha_y^m)\frac{1}{2}Cov\left(y_{t-1}^f, z_{t-1}^{G,f}\right) + \sigma^2_{z^{G,m}} + Cov\left(z_t^{C,m}, z_t^{G,m}\right)$$

and dividing by σ_{y^m} and $\sigma_{z^{G,m}}$ and rearranging we have

$$\begin{aligned} & \left(1-\frac{1}{2}\beta^m\alpha_y^m\right)\rho_{z^{G,m}y^m} - \frac{1}{2}\beta^m(1-\alpha_y^m)\frac{\sigma_{y^f}}{\sigma_{y^m}}\frac{\sigma_{z^{G,f}}}{\sigma_{z^{G,m}}}\rho_{z^{G,f}y^f} \\ = & \frac{\sigma_{z^{C,m}}}{\sigma_{y^m}}\rho_{z^{G,m}z^{C,m}} + \frac{1}{2}\beta^m\alpha_y^m\frac{\sigma_{z^{G,f}}}{\sigma_{z^{G,m}}}\rho_{y^m z^{G,f}} + \frac{1}{2}\beta^m(1-\alpha_y^m)\frac{\sigma_{y^f}}{\sigma_{y^m}}\rho_{z^{G,m}y^f} + \frac{\sigma_{z^{G,m}}}{\sigma_{y^m}} \end{aligned}$$

analogously we obtain another equation from the steady state assumption on $\rho_{z^{G,f}y^{,f}}$

$$\begin{aligned} & \left(1-\frac{1}{2}\beta^f\left(1-\alpha_y^f\right)\right)\rho_{z^{G,f}y^f} - \frac{1}{2}\beta^f\alpha_y^f\frac{\sigma_{y^m}}{\sigma_{y^f}}\frac{\sigma_{z^{G,m}}}{\sigma_{z^{G,f}}}\rho_{z^{G,m}y^m} \\ = & \frac{\sigma_{z^{C,f}}}{\sigma_{y^f}}\rho_{z^{G,f}z^{C,f}} + \frac{1}{2}\beta^f\left(1-\alpha_y^f\right)\frac{\sigma_{z^{G,m}}}{\sigma_{z^{G,f}}}\rho_{y^f z^{G,m}} + \frac{1}{2}\beta^f\alpha_y^f\frac{\sigma_{y^m}}{\sigma_{y^f}}\rho_{z^{G,f}y^m} + \frac{\sigma_{z^{G,f}}}{\sigma_{y^f}} \end{aligned}$$

We finally compute $\rho_{z^{C,m}y^m}$

$$\begin{aligned} Cov(y_t^m, z_t^{C,m}) &= Cov(\beta^m\widetilde{y}_{t-1}^m + z_t^{G,m} + z_t^{C,m}, z_t^{C,m}) \\ &= Cov\left(\beta^m\alpha_y^m y_{t-1}^m + \beta^m(1-\alpha_y^m)y_{t-1}^f, \gamma^m\alpha_z^k z_{t-1}^{C,m} + \gamma^m(1-\alpha_z^k)z_{t-1}^{C,f}\right) \\ &\quad + Cov\left(z_t^{C,m}, z_t^{G,m}\right) + \sigma^2_{z^{C,m}} \\ &= \beta^m\alpha_y^m\gamma^m\alpha_z^m Cov\left(y_{t-1}^m, z_{t-1}^{C,m}\right) + \beta^m\alpha_y^m\gamma^m(1-\alpha_z^k)Cov\left(y_{t-1}^m, z_{t-1}^{C,f}\right) \\ &\quad + \beta^m(1-\alpha_y^m)\gamma^m\alpha_z^k Cov\left(y_{t-1}^f, z_{t-1}^{C,m}\right) \\ &\quad + \beta^m(1-\alpha_y^m)\gamma^m(1-\alpha_z^k)Cov\left(y_{t-1}^f, z_{t-1}^{C,f}\right) + Cov\left(z_t^{C,m}, z_t^{G,m}\right) + \sigma^2_{z^{C,m}} \end{aligned}$$

and dividing by σ_{y^m} and $\sigma_{z^{C,m}}$ and rearranging we have

$$\begin{aligned} & \left(1-\beta^m\alpha_y^m\gamma^m\alpha_z^m\right)\rho_{z^{C,m}y^m} - \beta^m(1-\alpha_y^m)\gamma^m(1-\alpha_z^m)\frac{\sigma_{z^{C,f}}}{\sigma_{z^{C,m}}}\frac{\sigma_{y^f}}{\sigma_{y^m}}\rho_{z^{C,f}y^f} \\ = & \frac{\sigma_{z^{G,m}}}{\sigma_{y^m}}\rho_{z^{G,m}z^{C,m}} + \beta^m\alpha_y^m\gamma^m(1-\alpha_z^m)\frac{\sigma_{z^{C,f}}}{\sigma_{z^{C,m}}}\rho_{y^m z^{C,f}} + \beta^m(1-\alpha_y^m)\gamma^m\alpha_z^m\frac{\sigma_{y^f}}{\sigma_{y^m}}\rho_{z^{C,m}y^f} + \frac{\sigma_{z^{C,m}}}{\sigma_{y^m}} \end{aligned}$$

analogously we obtain another equation from the steady state assumption on $\rho_{z^{C,f}y^{,f}}$

$$\begin{aligned} & \left(1-\beta^f\left(1-\alpha_y^f\right)\gamma^f\alpha_z^f\right)\rho_{z^{C,f}y^f} - \beta^f\alpha_y^f\gamma^f\alpha_z^f\frac{\sigma_{z^{C,m}}}{\sigma_{z^{C,f}}}\frac{\sigma_{y^m}}{\sigma_{y^f}}\rho_{z^{C,m}y^m} \\ = & \frac{\sigma_{z^{G,f}}}{\sigma_{y^f}}\rho_{z^{G,f}z^{C,f}} + \beta^f\left(1-\alpha_y^f\right)\gamma^f\alpha_z^f\frac{\sigma_{z^{C,m}}}{\sigma_{z^{C,f}}}\rho_{y^f z^{C,m}} + \beta^f\alpha_y^f\gamma^f\left(1-\alpha_z^f\right)\frac{\sigma_{y^m}}{\sigma_{y^f}}\rho_{z^{C,f}y^m} + \frac{\sigma_{z^{C,f}}}{\sigma_{y^f}} \end{aligned}$$

The six equations for the steady state reduce the number of free correlations to four: $\rho_{z^{C,m}z^{C,f}}$, $\rho_{z^{C,m}y^f}$, $\rho_{y^m z^{C,f}}$, and $\rho_{y^m y^f}$. Then, this model has 21 parameters: $\beta^k, \gamma^k, \sigma_{z^{G,k}}, \sigma_{z^{C,k}}, \sigma^2_{x^k}, \sigma^2_{e^{C,k}}, \alpha_y^k, \alpha_z^k$, $k = m, f$, $\sigma_{x^m x^f}, \sigma_{e^{C,m}e^{C,f}}, \rho_{z^{C,m}z^{C,f}}$, $\rho_{z^{C,m}y^f}$, $\rho_{y^m z^{C,f}}$, and $\rho_{y^m y^f}$, just one parameter more than the one factor model.

M.3 Main Covariances

We first compute the main covariances (husband-wife, parent-child and siblings). Then, the covariances for other relatives are obtained recursively. We again use the notation in Figure 1 to denote individuals with different degrees of kinship.

Husband and wife $a - a'$

We have to compute the covariance between "a" and "a'". Let $n' = m, f$ be the gender of "a'" and $n = f, m$ the gender of "a"

$$Cov(y_{t-1}^{a,n}, y_{t-1}^{a',n'}) = \sigma_{y^m}\sigma_{y^f}\rho_{y^m y^f}$$

Parent–child $aa - a'$

We have to compute the covariance between "aa" and "a'". Let $n' = m, f$ be the gender of a' and $n^* = f, m$ the gender of aa. We project aa on a' (his/her father or mother).

$$\begin{aligned} Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,a',n'}) &= Cov(G_{zgn'}^{n^*} z_{t-1}^{G,a',n'} + G_{zn'}^{n^*} z_{t-1}^{C,a',n'} + G_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{G,a',n'}) \\ &= G_{zgn'}^{n^*}\sigma_{z^{G,n'}}^2 + G_{zn'}^{n^*} Cov(z_{t-1}^{C,a',n'}, z_{t-1}^{G,a',n'}) + G_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, z_{t-1}^{G,a',n'}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,a',n'}) &= Cov(G_{zgn'}^{n^*} z_{t-1}^{G,a',n'} + G_{zn'}^{n^*} z_{t-1}^{C,a',n'} + G_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{C,a',n'}) \\ &= G_{zgn'}^{n^*} Cov(z_{t-1}^{G,a',n'}, z_{t-1}^{C,a',n'}) + G_{zn'}^{n^*}\sigma_{z^{C,n'}}^2 + G_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, z_{t-1}^{C,a',n'}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{G,aa,n^*}, y_{t-1}^{a',n'}) &= Cov(G_{zgn'}^{n^*} z_{t-1}^{G,a',n'} + G_{zn'}^{n^*} z_{t-1}^{C,a',n'} + G_{yn'}^{n^*} y_{t-1}^{a',n'}, y_{t-1}^{a',n'}) \\ &= G_{zgn'}^{n^*} Cov(z_{t-1}^{G,a',n'}, y_{t-1}^{a',n'}) + G_{zn'}^{n^*} Cov(z_{t-1}^{C,a',n'}, y_{t-1}^{a',n'}) + G_{yn'}^{n^*}\sigma_{y^{n'}}^2 \end{aligned}$$

$$\begin{aligned} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,a',n'}) &= Cov(C_{zn'}^{n^*} z_{t-1}^{C,a',n'} + C_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{G,a',n'}) \\ &= C_{zn'}^{n^*} Cov(z_{t-1}^{C,a',n'}, z_{t-1}^{G,a',n'}) + C_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, z_{t-1}^{G,a',n'}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,a',n'}) &= Cov(C_{zn'}^{n^*} z_{t-1}^{C,a',n'} + C_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{C,a',n'}) \\ &= C_{zn'}^{n^*}\sigma_{z^{C,n'}}^2 + C_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, z_{t-1}^{C,a',n'}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{C,aa,n^*}, y_{t-1}^{a',n'}) &= Cov(C_{zn'}^{n^*} z_{t-1}^{C,a',n'} + C_{yn'}^{n^*} y_{t-1}^{a',n'}, y_{t-1}^{a',n'}) \\ &= C_{zn'}^{n^*} Cov(z_{t-1}^{C,a',n'}, y_{t-1}^{a',n'}) + C_{yn'}^{n^*}\sigma_{y^{n'}}^2 \end{aligned}$$

$$Cov(y_t^{aa,n^*}, z_{t-1}^{G,a',n'}) = Cov(B_{zgn'}^{n^*} z_{t-1}^{G,a',n'} + B_{zn'}^{n^*} z_{t-1}^{C,a',n'} + B_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{G,a',n'})$$
$$= B_{zgn'}^{n^*} \sigma^2_{z^{G,n'}} + B_{zn'}^{n^*} Cov(z_{t-1}^{C,a',n'}, z_{t-1}^{G,a',n'}) + B_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, z_{t-1}^{G,a',n'})$$

$$Cov(y_t^{aa,n^*}, z_{t-1}^{C,a',n'}) = Cov(B_{zgn'}^{n^*} z_{t-1}^{G,a',n'} + B_{zn'}^{n^*} z_{t-1}^{C,a',n'} + B_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{C,a',n'})$$
$$= B_{zgn'}^{n^*} Cov(z_{t-1}^{G,a',n'}, z_{t-1}^{C,a',n'}) + B_{zn'}^{n^*} \sigma^2_{z^{C,n'}} + B_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, z_{t-1}^{C,a',n'})$$

$$Cov(y_t^{aa,n^*}, y_{t-1}^{a',n'}) = Cov(B_{zgn'}^{n^*} z_{t-1}^{G,a',n'} + B_{zn'}^{n^*} z_{t-1}^{C,a',n'} + B_{yn'}^{n^*} y_{t-1}^{a',n'}, y_{t-1}^{a',n'})$$
$$= B_{zgn'}^{n^*} Cov(z_{t-1}^{G,a',n'}, y_{t-1}^{a',n'}) + B_{zn'}^{n^*} Cov(z_{t-1}^{C,a',n'}, y_{t-1}^{a',n'}) + B_{yn'}^{n^*} \sigma^2_{y^{n'}}$$

Siblings $a' - b$

We have to compute the covariance between "a'" and "b". Let $n', l = m, f$ be the genders of the siblings. We can compute the covariances projecting on the father ($k = m$) or on the mother ($k = f$).

$$Cov(z_t^{G,a',n'}, z_t^{G,b,l}) = G_{zgk}^{n'} G_{zgk}^{l} \sigma^2_{z^{G,k}} + G_{zk}^{n'} G_{zk}^{l} \sigma^2_{z^{C,k}} + G_{yk}^{n'} G_{yk}^{l} \sigma^2_{y^k} + \left(G_{zgk}^{n'} G_{zk}^{l} + G_{zk}^{n'} G_{zgk}^{l}\right) Cov(z_{t-1}^{G,a',k}, z_{t-1}^{C,a',k})$$
$$+ \left(G_{zgk}^{n'} G_{yk}^{l} + G_{yk}^{n'} G_{zgk}^{l}\right) Cov(z_{t-1}^{G,a',k}, y_{t-1}^{a',k}) + \left(G_{zk}^{n'} G_{yk}^{l} + G_{yk}^{n'} G_{zk}^{l}\right) Cov(z_{t-1}^{C,a',k}, y_{t-1}^{a',k}) + g_k^{n'} g_k^{l} \sigma^2_{w^{G,k}}$$

$$Cov(z_t^{G,a',n'}, z_t^{C,b,l}) = G_{zk}^{n'} C_{zk}^{l} \sigma^2_{z^{C,k}} + G_{yk}^{n'} C_{yk}^{l} \sigma^2_{y^k} + G_{zgk}^{n'} C_{zk}^{l} Cov(z_{t-1}^{G,a',k}, z_{t-1}^{C,a',k}) + G_{zgk}^{n'} C_{yk}^{l} Cov(z_{t-1}^{G,a',k}, y_{t-1}^{a',k})$$
$$+ \left(G_{zk}^{n'} C_{yk}^{l} + G_{yk}^{n'} C_{zk}^{l}\right) Cov(z_{t-1}^{C,a',k}, y_{t-1}^{a',k}) + g_k^{n'} c_k^{l} Cov(w_{t-1}^{G,k}, \omega_{t-1}^{C,k})$$

$$Cov(z_t^{G,a',n'}, y_t^{b,l}) = G_{zgk}^{n'} B_{zgk}^{l} \sigma^2_{z^{G,k}} + G_{zk}^{n'} B_{zk}^{l} \sigma^2_{z^{C,k}} + G_{yk}^{n'} B_{yk}^{l} \sigma^2_{y^k} + \left(G_{zgk}^{n'} B_{zk}^{l} + G_{zk}^{n'} B_{zgk}^{l}\right) Cov(z_{t-1}^{G,a',k}, z_{t-1}^{C,a',k})$$
$$+ \left(G_{zgk}^{n'} B_{yk}^{l} + G_{yk}^{n'} B_{zgk}^{l}\right) Cov(z_{t-1}^{G,a',k}, y_{t-1}^{a',k}) + \left(G_{zk}^{n'} B_{yk}^{l} + G_{yk}^{n'} B_{zk}^{l}\right) Cov(z_{t-1}^{C,a',k}, y_{t-1}^{a',k})$$
$$+ g_k^{n'} g_k^{l} \sigma^2_{w^{G,k}} + g_k^{n'} c_k^{l} Cov(w_{t-1}^{G,k}, \omega_{t-1}^{C,k}) + g_k^{n'} b_k^{l} Cov(w_{t-1}^{G,k}, \varepsilon_{t-1}^{k})$$

$$Cov(z_t^{C,a',n'}, z_t^{G,b,l}) = C_{zk}^{n'} G_{zk}^{l} \sigma^2_{z^{C,k}} + C_{yk}^{n'} G_{yk}^{l} \sigma^2_{y^k} + C_{zk}^{n'} G_{zgk}^{l} Cov(z_{t-1}^{G,a',k}, z_{t-1}^{C,a',k}) + C_{yk}^{n'} G_{zgk}^{l} Cov(z_{t-1}^{G,a',k}, y_{t-1}^{a',k})$$
$$+ \left(C_{zk}^{n'} G_{yk}^{l} + C_{yk}^{n'} G_{zk}^{l}\right) (z_{t-1}^{C,a',k}, y_{t-1}^{a',k}) + c_k^{n'} g_k^{l} Cov(w_{t-1}^{G,k}, \omega_{t-1}^{C,k})$$

$$Cov(z_t^{C,a',n'}, z_t^{C,b,l}) = c_{zk}^{n'} C_{zk}^{l} \sigma^2_{z^{C,k}} + c_{yk}^{n'} C_{yk}^{l} \sigma^2_{y^k} + \left(C_{zk}^{n'} C_{yk}^{l} + C_{yk}^{n'} C_{zk}^{l}\right) cov(z_{t-1}^{C,a',k}, y_{t-1}^{a',k}) + c_k^{n'} c_k^{l} \sigma^2_{w^{C,k}} + \sigma_{e^{c,n'} e^{c,l}}$$

$$Cov(z_t^{C,a',n'}, y_t^{b,l}) = C_{zk}^{n'} B_{zk}^{l} \sigma^2_{z^{C,k}} + C_{yk}^{n'} B_{yk}^{l} \sigma^2_{y^k} + C_{zk}^{n'} B_{zgk}^{l} Cov(z_{t-1}^{G,a',k}, z_{t-1}^{C,a',k}) + C_{yk}^{n'} B_{zgk}^{l} Cov(z_{t-1}^{G,a',k}, y_{t-1}^{a',k})$$
$$+ \left(C_{zk}^{n'} B_{yk}^{l} + C_{yk}^{n'} B_{zk}^{l}\right) cov(z_{t-1}^{C,a',k}, y_{t-1}^{a',k}) + c_k^{n'} g_k^{l} Cov(w_{t-1}^{C,k}, \omega_{t-1}^{G,k}) + c_k^{n'} c_k^{l} \sigma^2_{w^{C,k}} + c_k^{n'} b_k^{l} Cov(w_{t-1}^{C,k}, \varepsilon_{t-1}^{k}) + \sigma_{e^{c,n'} e^{c,l}}$$

$$Cov(y_t^{a',n'}, z_t^{G,b,l}) = B_{zgk}^{n'} G_{zgk}^{l} \sigma^2_{z^{G,k}} + B_{zk}^{n'} G_{zk}^{l} \sigma^2_{z^{C,k}} + B_{yk}^{n'} G_{yk}^{l} \sigma^2_{y^k} + \left(B_{zgk}^{n'} G_{zk}^{l} + B_{zk}^{n'} G_{zgk}^{l}\right) Cov(z_{t-1}^{G,a',k}, z_{t-1}^{C,a',k})$$
$$+ \left(B_{zgk}^{n'} G_{yk}^{l} + B_{yk}^{n'} G_{zgk}^{l}\right) Cov(z_{t-1}^{G,k}, y_{t-1}^{a',k}) + \left(B_{zk}^{n'} G_{yk}^{l} + B_{yk}^{n'} G_{zk}^{l}\right) Cov(z_{t-1}^{C,k}, y_{t-1}^{a',k})$$
$$+ g_m^{n'} g_k^{l} \sigma^2_{w^{G,k}} + c_k^{n'} g_k^{l} Cov(w_{t-1}^{C,m}, \omega_{t-1}^{G,k}) + b_m^{n'} g_k^{l} Cov(\varepsilon_{t-1}^{m}, \omega_{t-1}^{G,k})$$

$$Cov(y_t^{a',n'}, z_t^{C,b,l}) = B_{zk}^{n'} C_{zk}^{l} \sigma^2_{z^{C,k}} + B_{yk}^{n'} C_{yk}^{l} \sigma^2_{y^k} + B_{zgk}^{n'} C_{zk}^{l} Cov(z_{t-1}^{G,a',k}, z_{t-1}^{C,a',k}) + B_{zgk}^{n'} C_{yk}^{l} Cov(z_{t-1}^{G,a',k}, y_{t-1}^{a',k})$$
$$+ \left(B_{zk}^{n'} C_{yk}^{l} + B_{yk}^{n'} C_{zk}^{l}\right) Cov(z_{t-1}^{C,a',k}, y_{t-1}^{a',k}) + g_k^{n'} c_k^{l} Cov(w_{t-1}^{G,k}, \omega_{t-1}^{C,k})$$
$$+ c_k^{n'} c_k^{l} \sigma^2_{w^{C,k}} + b_k^{n'} c_k^{l} Cov(\varepsilon_{t-1}^{m}, \omega_{t-1}^{C,k}) + \sigma_{e^{c,n'} e^{c,l}}$$

$$Cov(y_t^{a',n'},y_t^{b,l}) = B_{zgk}^{n'} B_{zgk}^{l} \sigma^2_{z^{G,k}} + B_{zk}^{n'} B_{zk}^{l} \sigma^2_{z^{C,k}} + B_{yk}^{n'} B_{yk}^{l} \sigma^2_{y^k} + \left(B_{zgk}^{n'} B_{zk}^{l} + B_{zk}^{n'} B_{zgk}^{l}\right) Cov(z_{t-1}^{G,a',k}, z_{t-1}^{C,a',k})$$
$$+ \left(B_{zgk}^{n'} B_{yk}^{l} + B_{yk}^{n'} B_{zgk}^{l}\right) cov(z_{t-1}^{G,a',k}, y_{t-1}^{a',k}) + \left(B_{zk}^{n'} B_{yk}^{l} + B_{yk}^{n'} B_{zk}^{l}\right) cov(z_{t-1}^{C,a',k}, y_{t-1}^{a',k}) + b_k^{n'} b_k^l \sigma^2_{\varepsilon^k} + g_k^{n'} g_k^l \sigma^2_{w^{G,k}}$$
$$+ c_k^{n'} c_k^l \sigma^2_{w^{C,k}} + \left(g_k^{n'} c_k^l + c_k^{n'} g_k^l\right) Cov(\omega_{t-1}^{G,k}, \omega_{t-1}^{C,k}) + \left(b_k^{n'} g_k^l + g_k^{n'} b_k^l\right) Cov(\omega_{t-1}^{G,k}, \varepsilon_{t-1}^{m})$$
$$+ \left(b_k^{n'} c_k^l + c_k^{n'} b_k^l\right) Cov(\omega_{t-1}^{c,k}, \varepsilon_{t-1}^{m}) + \sigma_{e^{C,n'} e^{C,l}} + \sigma_{x^{n'} x^l}$$

M.4 Other covariances

Before we obtain the remaining covariances for different degrees of kinship we compute the linear projections of $z_{t-1}^{G,a',n'}$, $z_{t-1}^{C,a',n'}$ and $y_{t-1}^{a',n'}$ on $z_{t-1}^{G,b,l}$, $z_{t-1}^{C,b,l}$ and $y_{t-1}^{b,l}$, $n', l = m, f$, where a' and b are siblings.

$$\begin{aligned}
z_{t-1}^{G,a',n'} &= r_{z^G z^G}^{n',l} z_{t-1}^{G,b,l} + r_{z^G z^C}^{n',l} z_{t-1}^{C,b,l} + r_{z^G y}^{n',l} y_{t-1}^{b,l} + w_{t-1}^{G,n',l} \\
z_{t-1}^{C,a',n'} &= r_{z^C z^G}^{n',l} z_{t-1}^{G,b,l} + r_{z^C z^C}^{n',l} z_{t-1}^{C,b,l} + r_{z^C y}^{n',l} y_{t-1}^{b,l} + w_{t-1}^{C,n',l} \\
y_{t-1}^{a',n'} &= r_{yz^G}^{n',l} z_{t-1}^{G,b,l} + r_{yz^C}^{n',l} z_{t-1}^{C,b,l} + r_{yy}^{n',l} y_{t-1}^{b,l} + \varepsilon_{t-1}^{n',l}
\end{aligned}$$

where $w_{t-1}^{G,n',l}$, $w_{t-1}^{C,n',l}$ and $\varepsilon_{t-1}^{n',l}$ might be correlated but are uncorrelated with $z_{t-1}^{G,b,l}$, $z_{t-1}^{C,b,l}$ and $y_{t-1}^{b,l}$. We have that

$$\begin{pmatrix} r_{z^G z^G}^{n',l} & r_{z^G z^C}^{n',l} & r_{z^G y}^{n',l} \\ r_{z^C z^G}^{n',l} & r_{z^C z^C}^{n',l} & r_{z^C y}^{n',l} \\ r_{yz^G}^{n',l} & r_{yz^C}^{n',l} & r_{yy}^{n',l} \end{pmatrix}' = \begin{pmatrix} \sigma^2_{z^{G,l}} & \sigma_{z^{G,l} z^{C,l}} & \sigma_{z^{G,l} y^l} \\ \sigma_{z^{G,l} z^{C,l}} & \sigma^2_{z^{C,l}} & \sigma_{z^{C,l} y^l} \\ \sigma_{z^{G,l} y^l} & \sigma_{z^{C,l} y^l} & \sigma^2_{y^l} \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{z^{G,b,l} z^{G,a',n'}} & \sigma_{z^{G,b,l} z^{C,a',n'}} & \sigma_{z^{G,b,l} y^{a',n'}} \\ \sigma_{z^{C,b,l} z^{G,a',n'}} & \sigma_{z^{C,b,l} z^{C,a',n'}} & \sigma_{z^{C,b,l} y^{a',n'}} \\ \sigma_{y^{b,l} z^{G,a',n'}} & \sigma_{y^{b,l} z^{C,a',n'}} & \sigma_{y^{b,l} y^{a',n'}} \end{pmatrix}$$

Consanguine relatives ("blood")

Vertical covariances

Uncle/aunt (siblings of the parents) $aa - b$

We have to compute the covariances between "aa" and "b". Let $n^* = m, f$ be the gender of aa and $l = m, f$ the gender of b. We project aa on a' (his/her father or mother) who has gender n'

$$\begin{aligned}
Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b,l}) &= Cov(G_{zgn'}^{n^*} z_{t-1}^{G,a',n'} + G_{zn'}^{n^*} z_{t-1}^{C,a',n'} + G_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{G,b,l}) = \\
&\quad G_{zgn'}^{n^*} Cov(z_{t-1}^{G,a',n'}, z_{t-1}^{G,b,l}) + G_{zn'}^{n^*} Cov(z_{t-1}^{C,a',n'}, z_{t-1}^{G,b,l}) + G_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, z_{t-1}^{G,b,l})
\end{aligned}$$

$$\begin{aligned}
Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b,l}) &= Cov(G_{zgn'}^{n^*} z_{t-1}^{G,a',n'} + G_{zn'}^{n^*} z_{t-1}^{C,a',n'} + G_{yn'}^{n^*} y_{t-1}^{a',n'}, z_{t-1}^{C,b,l}) = \\
&\quad G_{zgn'}^{n^*} Cov(z_{t-1}^{G,a',n'}, z_{t-1}^{C,b,l}) + G_{zn'}^{n^*} Cov(z_{t-1}^{C,a',n'}, z_{t-1}^{C,b,l}) + G_{yn'}^{n^*} Cov(y_{t-1}^{a',n'}, z_{t-1}^{C,b,l})
\end{aligned}$$

$$Cov(z_t^{G,aa,n^*}, y_{t-1}^{b,l}) = Cov(G_{zgn'}^{n^*} z_{t-1}^{G,a',n'} + G_{zn'}^{n^*} z_{t-1}^{C,a',n'} + G_{yn'}^{n^*} y_{t-1}^{a',n'}, y_{t-1}^{b,l}) =$$

$$G^{n^*}_{zgn'}Cov(z^{G,a',n'}_{t-1}, y^{b,l}_{t-1}) + G^{n^*}_{zn'}Cov(z^{C,a',n'}_{t-1}, y^{b,l}_{t-1}) + G^{n^*}_{yn'}Cov(y^{a',n'}_{t-1}, y^{b,l}_{t-1})$$

$$\begin{aligned}
Cov(z^{C,aa,n^*}_t, z^{G,b,l}_{t-1}) &= Cov(C^{n^*}_{zgn'}z^{G,a',n'}_{t-1} + C^{n^*}_{zn'}z^{C,a',n'}_{t-1} + C^{n^*}_{yn'}y^{a',n'}_{t-1}, z^{G,b,l}_{t-1}) = \\
& C^{n^*}_{zgn'}Cov(z^{G,a',n'}_{t-1}, z^{G,b,l}_{t-1}) + C^{n^*}_{zn'}Cov(z^{C,a',n'}_{t-1}, z^{G,b,l}_{t-1}) + C^{n^*}_{yn'}Cov(y^{a',n'}_{t-1}, z^{G,b,l}_{t-1})
\end{aligned}$$

$$\begin{aligned}
Cov(z^{C,aa,n^*}_t, z^{C,b,l}_{t-1}) &= Cov(C^{n^*}_{zgn'}z^{G,a',n'}_{t-1} + C^{n^*}_{zn'}z^{C,a',n'}_{t-1} + C^{n^*}_{yn'}y^{a',n'}_{t-1}, z^{C,b,l}_{t-1}) = \\
& C^{n^*}_{zgn'}Cov(z^{G,a',n'}_{t-1}, z^{C,b,l}_{t-1}) + C^{n^*}_{zn'}Cov(z^{C,a',n'}_{t-1}, z^{C,b,l}_{t-1}) + C^{n^*}_{yn'}Cov(y^{a',n'}_{t-1}, z^{C,b,l}_{t-1})
\end{aligned}$$

$$\begin{aligned}
Cov(z^{C,aa,n^*}_t, y^{b,l}_{t-1}) &= Cov(C^{n^*}_{zgn'}z^{G,a',n'}_{t-1} + C^{n^*}_{zn'}z^{C,a',n'}_{t-1} + C^{n^*}_{yn'}y^{a',n'}_{t-1}, y^{b,l}_{t-1}) = \\
& C^{n^*}_{zgn'}Cov(z^{G,a',n'}_{t-1}, y^{b,l}_{t-1}) + C^{n^*}_{zn'}Cov(z^{C,a',n'}_{t-1}, y^{b,l}_{t-1}) + C^{n^*}_{yn'}Cov(y^{a',n'}_{t-1}, y^{b,l}_{t-1})
\end{aligned}$$

$$\begin{aligned}
Cov(y^{aa,n^*}_t, z^{G,b,l}_{t-1}) &= Cov(B^{n^*}_{zgn'}z^{G,a',n'}_{t-1} + B^{n^*}_{zn'}z^{C,a',n'}_{t-1} + B^{n^*}_{yn'}y^{a',n'}_{t-1}, z^{G,b,l}_{t-1}) = \\
& B^{n^*}_{zgn'}Cov(z^{G,a',n'}_{t-1}, z^{G,b,l}_{t-1}) + B^{n^*}_{zn'}Cov(z^{C,a',n'}_{t-1}, z^{G,b,l}_{t-1}) + B^{n^*}_{yn'}Cov(y^{a',n'}_{t-1}, z^{G,b,l}_{t-1})
\end{aligned}$$

$$\begin{aligned}
Cov(y^{aa,n^*}_t, z^{C,b,l}_{t-1}) &= Cov(B^{n^*}_{zgn'}z^{G,a',n'}_{t-1} + B^{n^*}_{zn'}z^{C,a',n'}_{t-1} + B^{n^*}_{yn'}y^{a',n'}_{t-1}, z^{C,b,l}_{t-1}) = \\
& B^{n^*}_{zgn'}Cov(z^{G,a',n'}_{t-1}, z^{C,b,l}_{t-1}) + B^{n^*}_{zn'}Cov(z^{C,a',n'}_{t-1}, z^{C,b,l}_{t-1}) + B^{n^*}_{yn'}Cov(y^{a',n'}_{t-1}, z^{C,b,l}_{t-1})
\end{aligned}$$

$$\begin{aligned}
Cov(y^{aa,n^*}_t, y^{b,l}_{t-1}) &= Cov(B^{n^*}_{zgn'}z^{G,a',n'}_{t-1} + B^{n^*}_{zn'}z^{C,a',n'}_{t-1} + B^{n^*}_{yn'}y^{a',n'}_{t-1}, y^{b,l}_{t-1}) = \\
& B^{n^*}_{zgn'}Cov(z^{G,a',n'}_{t-1}, y^{b,l}_{t-1}) + B^{n^*}_{zn'}Cov(z^{C,a',n'}_{t-1}, y^{b,l}_{t-1}) + B^{n^*}_{yn'}Cov(y^{a',n'}_{t-1}, y^{b,l}_{t-1})
\end{aligned}$$

where a' and b are siblings.

Horizontal covariances

Cousins $aa - bb$

We have to compute the covariances between "aa" and "bb". Let $n^* = m, f$ be the gender of aa and $l^* = m, f$ the gender of the bb. We project bb on b (his/her father or mother) who has gender l

$$\begin{aligned}
Cov(z^{G,aa,n^*}_t, z^{G,bb,l^*}_t) &= Cov(z^{G,aa,n^*}_t, G^{l^*}_{zgl}z^{G,b,l}_{t-1} + G^{l^*}_{zl}z^{C,b,l}_{t-1} + G^{l^*}_{yl}y^{b,l}_{t-1}) \\
&= G^{l^*}_{zgl}Cov(z^{G,aa,n^*}_t, z^{G,b,l}_{t-1}) + G^{l^*}_{zl}Cov(z^{G,aa,n^*}_t, z^{C,b,l}_{t-1}) + G^{l^*}_{yl}Cov(z^{G,aa,n^*}_t, y^{b,l}_{t-1})
\end{aligned}$$

$$\begin{aligned}
Cov(z^{G,aa,n^*}_t, z^{C,bb,l^*}_t) &= Cov(z^{G,aa,n^*}_t, C^{l^*}_{zgl}z^{G,b,l}_{t-1} + C^{l^*}_{zl}z^{C,b,l}_{t-1} + C^{l^*}_{yl}y^{b,l}_{t-1}) \\
&= C^{l^*}_{zgl}Cov(z^{G,aa,n^*}_t, z^{G,b,l}_{t-1}) + C^{l^*}_{zl}Cov(z^{G,aa,n^*}_t, z^{C,b,l}_{t-1}) + C^{l^*}_{yl}Cov(z^{G,aa,n^*}_t, y^{b,l}_{t-1})
\end{aligned}$$

$$\begin{aligned} Cov(z_t^{G,aa,n^*}, y_t^{bb,l^*}) &= Cov(z_t^{G,aa,n^*}, B_{zgl}^{l^*} z_{t-1}^{G,b,l} + B_{zl}^{l^*} z_{t-1}^{C,b,l} + B_{yl}^{l^*} y_{t-1}^{b,l}) \\ &= B_{zgl}^{l^*} Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b,l}) + B_{zl}^{l^*} Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b,l}) + B_{yl}^{l^*} Cov(z_t^{G,aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{C,aa,n^*}, z_t^{G,bb,l^*}) &= Cov(z_t^{C,aa,n^*}, G_{zgl}^{l^*} z_{t-1}^{G,b,l} + G_{zl}^{l^*} z_{t-1}^{C,b,l} + G_{yl}^{l^*} y_{t-1}^{b,l}) \\ &= G_{zgl}^{l^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b,l}) + G_{zl}^{l^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b,l}) + G_{yl}^{l^*} Cov(z_t^{C,aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{C,aa,n^*}, z_t^{C,bb,l^*}) &= Cov(z_t^{C,aa,n^*}, C_{zgl}^{l^*} z_{t-1}^{G,b,l} + C_{zl}^{l^*} z_{t-1}^{C,b,l} + C_{yl}^{l^*} y_{t-1}^{b,l}) \\ &= C_{zgl}^{l^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b,l}) + C_{zl}^{l^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b,l}) + C_{yl}^{l^*} Cov(z_t^{C,aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{C,aa,n^*}, y_t^{bb,l^*}) &= Cov(z_t^{C,aa,n^*}, B_{zgl}^{l^*} z_{t-1}^{G,b,l} + B_{zl}^{l^*} z_{t-1}^{C,b,l} + B_{yl}^{l^*} y_{t-1}^{b,l}) \\ &= B_{zgl}^{l^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b,l}) + B_{zl}^{l^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b,l}) + B_{yl}^{l^*} Cov(z_t^{C,aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(y_t^{aa,n^*}, z_t^{G,bb,l^*}) &= Cov(y_t^{aa,n^*}, G_{zgl}^{l^*} z_{t-1}^{G,b,l} + G_{zl}^{l^*} z_{t-1}^{C,b,l} + G_{yl}^{l^*} y_{t-1}^{b,l}) \\ &= G_{zgl}^{l^*} Cov(y_t^{aa,n^*}, z_{t-1}^{G,b,l}) + G_{zl}^{l^*} Cov(y_t^{aa,n^*}, z_{t-1}^{C,b,l}) + G_{yl}^{l^*} Cov(y_t^{aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{G,aa,n^*}, z_t^{C,bb,l^*}) &= Cov(y_t^{aa,n^*}, C_{zgl}^{l^*} z_{t-1}^{G,b,l} + C_{zl}^{l^*} z_{t-1}^{C,b,l} + C_{yl}^{l^*} y_{t-1}^{b,l}) \\ &= C_{zgl}^{l^*} Cov(y_t^{aa,n^*}, z_{t-1}^{G,b,l}) + C_{zl}^{l^*} Cov(y_t^{aa,n^*}, z_{t-1}^{C,b,l}) + C_{yl}^{l^*} Cov(y_t^{aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(y_t^{aa,n^*}, y_t^{bb,l^*}) &= Cov(y_t^{aa,n^*}, B_{zgl}^{l^*} z_{t-1}^{G,b,l} + B_{zl}^{l^*} z_{t-1}^{C,b,l} + B_{yl}^{l^*} y_{t-1}^{b,l}) \\ &= B_{zgl}^{l^*} Cov(y_t^{aa,n^*}, z_{t-1}^{G,b,l}) + B_{zl}^{l^*} Cov(y_t^{aa,n^*}, z_{t-1}^{C,b,l}) + B_{yl}^{l^*} Cov(y_t^{aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

where b is the uncle/aunt of aa.

Affinity relatives ("in-law")

Vertical covariances

Spouse of the uncle/aunt (spouses of the siblings of the parents) $aa - b'$

We have to compute the covariances between "aa" and "b'". Let $n^* = m, f$ be the gender of aa and $l' = m, f$ the gender of the b'. We project b' on his/her spouse b

$$Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b',l'}) = Cov(z_t^{G,aa,n^*}, r_{z^G z^G}^{l} z_{t-1}^{G,b,l} + r_{z^G z^C}^{l} z_{t-1}^{C,b,l} + r_{z^G y}^{l} y_{t-1}^{b,l})$$

$$= r^l_{z^Gz^G}Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b,l}) + r^l_{z^Gz^C}Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b,l}) + r^l_{z^Gy}Cov(z_t^{G,aa,n^*}, y_{t-1}^{b,l})$$

$$\begin{aligned} Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b',l'}) &= Cov(z_t^{G,aa,n^*}, r^l_{z^Cz^G}z_{t-1}^{G,b,l} + r^l_{z^Cz^C}z_{t-1}^{C,b,l} + r^l_{z^Cy}y_{t-1}^{b,l}) \\ &= r^l_{z^Cz^G}Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b,l}) + r^l_{z^Cz^C}Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b,l}) + r^l_{z^Cy}Cov(z_t^{G,aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{G,aa,n^*}, y_{t-1}^{b',l'}) &= Cov(z_t^{G,aa,n^*}, r^l_{yz^G}z_{t-1}^{G,b,l} + r^l_{yz^C}z_{t-1}^{C,b,l} + r^l_{yy}y_{t-1}^{b,l}) \\ &= r^l_{yz^G}Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b,l}) + r^l_{yz^C}Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b,l}) + r^l_{yy}Cov(z_t^{G,aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b',l'}) &= Cov(z_t^{C,aa,n^*}, r^l_{z^Gz^G}z_{t-1}^{G,b,l} + r^l_{z^Gz^C}z_{t-1}^{C,b,l} + r^l_{z^Gy}y_{t-1}^{b,l}) \\ &= r^l_{z^Gz^G}Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b,l}) + r^l_{z^Gz^C}Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b,l}) + r^l_{z^Gy}Cov(z_t^{C,aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b',l'}) &= Cov(z_t^{C,aa,n^*}, r^l_{z^Cz^G}z_{t-1}^{G,b,l} + r^l_{z^Cz^C}z_{t-1}^{C,b,l} + r^l_{z^Cy}y_{t-1}^{b,l}) \\ &= r^l_{z^Cz^G}Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b,l}) + r^l_{z^Cz^C}Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b,l}) + r^l_{z^Cy}Cov(z_t^{C,aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(z_t^{C,aa,n^*}, y_{t-1}^{b',l'}) &= Cov(z_t^{C,aa,n^*}, r^l_{yz^G}z_{t-1}^{G,b,l} + r^l_{yz^C}z_{t-1}^{C,b,l} + r^l_{yy}y_{t-1}^{b,l}) \\ &= r^l_{yz^G}Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b,l}) + r^l_{yz^C}Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b,l}) + r^l_{yy}Cov(z_t^{C,aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(y_t^{aa,n^*}, z_{t-1}^{G,b',l'}) &= Cov(y_t^{aa,n^*}, r^l_{z^Gz^G}z_{t-1}^{G,b,l} + r^l_{z^Gz^C}z_{t-1}^{C,b,l} + r^l_{z^Gy}y_{t-1}^{b,l}) \\ &= r^l_{z^Gz^G}Cov(y_t^{aa,n^*}, z_{t-1}^{G,b,l}) + r^l_{z^Gz^C}Cov(y_t^{aa,n^*}, z_{t-1}^{C,b,l}) + r^l_{z^Gy}Cov(y_t^{aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(y_t^{aa,n^*}, z_{t-1}^{C,b',l'}) &= Cov(y_t^{aa,n^*}, r^l_{z^Cz^G}z_{t-1}^{G,b,l} + r^l_{z^Cz^C}z_{t-1}^{C,b,l} + r^l_{z^Cy}y_{t-1}^{b,l}) \\ &= r^l_{z^Cz^G}Cov(y_t^{aa,n^*}, z_{t-1}^{G,b,l}) + r^l_{z^Cz^C}Cov(y_t^{aa,n^*}, z_{t-1}^{C,b,l}) + r^l_{z^Cy}Cov(y_t^{aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

$$\begin{aligned} Cov(y_t^{aa,n^*}, y_{t-1}^{b',l'}) &= Cov(y_t^{aa,n^*}, r^l_{yz^G}z_{t-1}^{G,b,l} + r^l_{yz^C}z_{t-1}^{C,b,l} + r^l_{yy}y_{t-1}^{b,l}) \\ &= r^l_{yz^G}Cov(y_t^{aa,n^*}, z_{t-1}^{G,b,l}) + r^l_{yz^C}Cov(y_t^{aa,n^*}, z_{t-1}^{C,b,l}) + r^l_{yy}Cov(y_t^{aa,n^*}, y_{t-1}^{b,l}) \end{aligned}$$

where b is uncle/aunt of aa.

Siblings of the siblings in law of the parents $aa - c$

We have to compute the covariances between "aa" and "c". Let $n^* = m, f$ be the gender of aa and $o = m, f$

the gender of the c. We project c on his/her sibling b'

$$\begin{aligned}Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,c,o}) &= Cov(z_t^{G,aa,n^*}, r_{z^G z^G}^{o,l'} z_{t-1}^{G,b',l'} + r_{z^G z^C}^{o,l'} z_{t-1}^{C,b',l'} + r_{z^G y}^{o,l'} y_{t-1}^{b',l'}) \\ &= r_{z^G z^G}^{o,l'} Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b',l'}) + r_{z^G z^C}^{o,l'} Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b',l'}) + r_{z^G y}^{o,l'} Cov(z_t^{G,aa,n^*}, y_{t-1}^{b',l'})\end{aligned}$$

$$\begin{aligned}Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,c,o}) &= Cov(z_t^{G,aa,n^*}, r_{z^C z^G}^{o,l'} z_{t-1}^{G,b',l'} + r_{z^C z^C}^{o,l'} z_{t-1}^{C,b',l'} + r_{z^C y}^{o,l'} y_{t-1}^{b',l'}) \\ &= r_{z^C z^G}^{o,l'} Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b',l'}) + r_{z^C z^C}^{o,l'} Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b',l'}) + r_{z^C y}^{o,l'} Cov(z_t^{G,aa,n^*}, y_{t-1}^{b',l'})\end{aligned}$$

$$\begin{aligned}Cov(z_t^{G,aa,n^*}, y_{t-1}^{c,o}) &= Cov(z_t^{G,aa,n^*}, r_{yz^G}^{o,l'} z_{t-1}^{G,b',l'} + r_{yz^C}^{o,l'} z_{t-1}^{C,b',l'} + r_{yy}^{o,l'} y_{t-1}^{b',l'}) \\ &= r_{yz^G}^{o,l'} Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b',l'}) + r_{yz^C}^{o,l'} Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b',l'}) + r_{yy}^{o,l'} Cov(z_t^{G,aa,n^*}, y_{t-1}^{b',l'})\end{aligned}$$

$$\begin{aligned}Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,c,o}) &= Cov(z_t^{C,aa,n^*}, r_{z^G z^G}^{o,l'} z_{t-1}^{G,b',l'} + r_{z^G z^C}^{o,l'} z_{t-1}^{C,b',l'} + r_{z^G y}^{o,l'} y_{t-1}^{b',l'}) \\ &= r_{z^G z^G}^{o,l'} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b',l'}) + r_{z^G z^C}^{o,l'} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b',l'}) + r_{z^G y}^{o,l'} Cov(z_t^{C,aa,n^*}, y_{t-1}^{b',l'})\end{aligned}$$

$$\begin{aligned}Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,c,o}) &= Cov(z_t^{C,aa,n^*}, r_{z^C z^G}^{o,l'} z_{t-1}^{G,b',l'} + r_{z^C z^C}^{o,l'} z_{t-1}^{C,b',l'} + r_{z^C y}^{o,l'} y_{t-1}^{b',l'}) \\ &= r_{z^C z^G}^{o,l'} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b',l'}) + r_{z^C z^C}^{o,l'} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b',l'}) + r_{z^C y}^{o,l'} Cov(z_t^{C,aa,n^*}, y_{t-1}^{b',l'})\end{aligned}$$

$$\begin{aligned}Cov(z_t^{C,aa,n^*}, y_{t-1}^{c,o}) &= Cov(z_t^{C,aa,n^*}, r_{yz^G}^{o,l'} z_{t-1}^{G,b',l'} + r_{yz^C}^{o,l'} z_{t-1}^{C,b',l'} + r_{yy}^{o,l'} y_{t-1}^{b',l'}) \\ &= r_{yz^G}^{o,l'} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b',l'}) + r_{yz^C}^{o,l'} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b',l'}) + r_{yy}^{o,l'} Cov(z_t^{C,aa,n^*}, y_{t-1}^{b',l'})\end{aligned}$$

$$\begin{aligned}Cov(y_t^{aa,n^*}, z_{t-1}^{G,c,o}) &= Cov(y_t^{aa,n^*}, r_{z^G z^G}^{o,l'} z_{t-1}^{G,b',l'} + r_{z^G z^C}^{o,l'} z_{t-1}^{C,b',l'} + r_{z^G y}^{o,l'} y_{t-1}^{b',l'}) \\ &= r_{z^G z^G}^{o,l'} Cov(y_t^{aa,n^*}, z_{t-1}^{G,b',l'}) + r_{z^G z^C}^{o,l'} Cov(y_t^{aa,n^*}, z_{t-1}^{C,b',l'}) + r_{z^G y}^{o,l'} Cov(y_t^{aa,n^*}, y_{t-1}^{b',l'})\end{aligned}$$

$$\begin{aligned}Cov(y_t^{aa,n^*}, z_{t-1}^{C,c,o}) &= Cov(y_t^{aa,n^*}, r_{z^C z^G}^{o,l'} z_{t-1}^{G,b',l'} + r_{z^C z^C}^{o,l'} z_{t-1}^{C,b',l'} + r_{z^C y}^{o,l'} y_{t-1}^{b',l'}) \\ &= r_{z^C z^G}^{o,l'} Cov(y_t^{aa,n^*}, z_{t-1}^{G,b',l'}) + r_{z^C z^C}^{o,l'} Cov(y_t^{aa,n^*}, z_{t-1}^{C,b',l'}) + r_{z^C y}^{o,l'} Cov(y_t^{aa,n^*}, y_{t-1}^{b',l'})\end{aligned}$$

$$\begin{aligned}Cov(y_t^{aa,n^*}, y_{t-1}^{c,o}) &= Cov(y_t^{aa,n^*}, r_{yz^G}^{o,l'} z_{t-1}^{G,b',l'} + r_{yz^C}^{o,l'} z_{t-1}^{C,b',l'} + r_{yy}^{o,l'} y_{t-1}^{b',l'}) \\ &= r_{yz^G}^{o,l'} Cov(y_t^{aa,n^*}, z_{t-1}^{G,b',l'}) + r_{yz^C}^{o,l'} Cov(y_t^{aa,n^*}, z_{t-1}^{C,b',l'}) + r_{yy}^{o,l'} Cov(y_t^{aa,n^*}, y_{t-1}^{b',l'})\end{aligned}$$

where y is the spouse of the uncle/aunt of aa.

We can recursively compute the covariance for spouses of the siblings in law of the parents and siblings of the siblings in law of the parents of any degree.

Horizontal covariances

Siblings in law $a-b$

We have to compute the covariances between "a" and "b". Let $n=m,f$ be the gender of a and $l=m,f$ the gender of the b. We project a on his/her spouse a' with gender $n'=f,m$

$$\begin{aligned}Cov(z_{t-1}^{G,a,n},z_{t-1}^{G,b,l}) &= Cov(r_{z^Gz^G}^{n'}z_{t-1}^{G,a',n'}+r_{z^Gz^C}^{n'}z_{t-1}^{C,a',n'}+r_{z^Gy}^{n'}y_{t-1}^{a',n'},z_{t-1}^{G,b,l})\\ &= r_{z^Gz^G}^{n'}Cov(z_{t-1}^{G,a',n'},z_{t-1}^{G,b,l})+r_{z^Gz^C}^{n'}Cov(z_{t-1}^{C,a',n'},z_{t-1}^{G,b,l})+r_{z^Gy}^{n'}Cov(y_{t-1}^{a',n'},z_{t-1}^{G,b,l})\end{aligned}$$

$$\begin{aligned}Cov(z_{t-1}^{G,a,n},z_{t-1}^{C,b,l}) &= Cov(r_{z^Gz^G}^{n'}z_{t-1}^{G,a',n'}+r_{z^Gz^C}^{n'}z_{t-1}^{C,a',n'}+r_{z^Gy}^{n'}y_{t-1}^{a',n'},z_{t-1}^{C,b,l})\\ &= r_{z^Gz^G}^{n'}Cov(z_{t-1}^{G,a',n'},z_{t-1}^{C,b,l})+r_{z^Gz^C}^{n'}Cov(z_{t-1}^{C,a',n'},z_{t-1}^{C,b,l})+r_{z^Gy}^{n'}Cov(y_{t-1}^{a',n'},z_{t-1}^{C,b,l})\end{aligned}$$

$$\begin{aligned}Cov(z_{t-1}^{G,a,n},y_{t-1}^{b,l}) &= Cov(r_{z^Gz^G}^{n'}z_{t-1}^{G,a',n'}+r_{z^Gz^C}^{n'}z_{t-1}^{C,a',n'}+r_{z^Gy}^{n'}y_{t-1}^{a',n'},y_{t-1}^{b,l})\\ &= r_{z^Gz^G}^{n'}Cov(z_{t-1}^{G,a',n'},y_{t-1}^{b,l})+r_{z^Gz^C}^{n'}Cov(z_{t-1}^{C,a',n'},y_{t-1}^{b,l})+r_{z^Gy}^{n'}Cov(y_{t-1}^{a',n'},y_{t-1}^{b,l})\end{aligned}$$

$$\begin{aligned}Cov(z_{t-1}^{C,a,n},z_{t-1}^{G,b,l}) &= Cov(r_{z^Cz^G}^{n'}z_{t-1}^{G,a',n'}+r_{z^Cz^C}^{n'}z_{t-1}^{C,a',n'}+r_{z^Cy}^{n'}y_{t-1}^{a',n'},z_{t-1}^{G,b,l})\\ &= r_{z^Cz^G}^{n'}Cov(z_{t-1}^{G,a',n'},z_{t-1}^{G,b,l})+r_{z^Cz^C}^{n'}Cov(z_{t-1}^{C,a',n'},z_{t-1}^{G,b,l})+r_{z^Cy}^{n'}Cov(y_{t-1}^{a',n'},z_{t-1}^{G,b,l})\end{aligned}$$

$$\begin{aligned}Cov(z_{t-1}^{C,a,n},z_{t-1}^{C,b,l}) &= Cov(r_{z^Cz^G}^{n'}z_{t-1}^{G,a',n'}+r_{z^Cz^C}^{n'}z_{t-1}^{C,a',n'}+r_{z^Cy}^{n'}y_{t-1}^{a',n'},z_{t-1}^{C,b,l})\\ &= r_{z^Cz^G}^{n'}Cov(z_{t-1}^{G,a',n'},z_{t-1}^{C,b,l})+r_{z^Cz^C}^{n'}Cov(z_{t-1}^{C,a',n'},z_{t-1}^{C,b,l})+r_{z^Cy}^{n'}Cov(y_{t-1}^{a',n'},z_{t-1}^{C,b,l})\end{aligned}$$

$$\begin{aligned}Cov(z_{t-1}^{C,a,n},y_{t-1}^{b,l}) &= Cov(r_{z^Cz^G}^{n'}z_{t-1}^{G,a',n'}+r_{z^Cz^C}^{n'}z_{t-1}^{C,a',n'}+r_{z^Cy}^{n'}y_{t-1}^{a',n'},y_{t-1}^{b,l})\\ &= r_{z^Cz^G}^{n'}Cov(z_{t-1}^{G,a',n'},y_{t-1}^{b,l})+r_{z^Cz^C}^{n'}Cov(z_{t-1}^{C,a',n'},y_{t-1}^{b,l})+r_{z^Cy}^{n'}Cov(y_{t-1}^{a',n'},y_{t-1}^{b,l})\end{aligned}$$

$$\begin{aligned}Cov(y_{t-1}^{a,n},z_{t-1}^{G,b,l}) &= Cov(r_{yz^G}^{n'}z_{t-1}^{G,a',n'}+r_{yz^C}^{n'}z_{t-1}^{C,a',n'}+r_{yy}^{n'}y_{t-1}^{a',n'},z_{t-1}^{G,b,l})\\ &= r_{yz^G}^{n'}Cov(z_{t-1}^{G,a',n'},z_{t-1}^{G,b,l})+r_{yz^C}^{n'}Cov(z_{t-1}^{C,a',n'},z_{t-1}^{G,b,l})+r_{yy}^{n'}Cov(y_{t-1}^{a',n'},z_{t-1}^{G,b,l})\end{aligned}$$

$$\begin{aligned}Cov(y_{t-1}^{a,n},z_{t-1}^{C,b,l}) &= Cov(r_{yz^G}^{n'}z_{t-1}^{G,a',n'}+r_{yz^C}^{n'}z_{t-1}^{C,a',n'}+r_{yy}^{n'}y_{t-1}^{a',n'},z_{t-1}^{C,b,l})\\ &= r_{yz^G}^{n'}Cov(z_{t-1}^{G,a',n'},z_{t-1}^{C,b,l})+r_{yz^C}^{n'}Cov(z_{t-1}^{C,a',n'},z_{t-1}^{C,b,l})+r_{yy}^{n'}Cov(y_{t-1}^{a',n'},z_{t-1}^{C,b,l})\end{aligned}$$

$$
\begin{aligned}
Cov(y_{t-1}^{a,n}, y_{t-1}^{b,l}) &= Cov(r_{yz^G}^{n'} z_{t-1}^{G,a',n'} + r_{yz^C}^{n'} z_{t-1}^{C,a',n'} + r_{yy}^{n'} y_{t-1}^{a',n'}, y_{t-1}^{b,l}) \\
&= r_{yz^G}^{n'} Cov(z_{t-1}^{G,a',n'}, y_{t-1}^{b,l}) + r_{yz^C}^{n'} Cov(z_{t-1}^{C,a',n'}, y_{t-1}^{b,l}) + r_{yy}^{n'} Cov(y_{t-1}^{a',n'}, y_{t-1}^{b,l})
\end{aligned}
$$

where a' and b are siblings.

Spouse of the siblings in law $a - b'$

We have to compute the covariances between "a" and "b'". Let $n = m, f$ be the gender of a and $l' = m, f$ the gender of the b'. We project b' on his/her spouse b

$$
\begin{aligned}
Cov(z_t^{G,a,n}, z_{t-1}^{G,b',l'}) &= Cov(z_t^{G,a,n}, r_{z^G z^G}^{l} z_{t-1}^{G,b,l} + r_{z^G z^C}^{l} z_{t-1}^{C,b,l} + r_{z^G y}^{l} y_{t-1}^{b,l}) \\
&= r_{z^G z^G}^{l} Cov(z_t^{G,a,n}, z_{t-1}^{G,b,l}) + r_{z^G z^C}^{l} Cov(z_t^{G,a,n}, z_{t-1}^{C,b,l}) + r_{z^G y}^{l} Cov(z_t^{G,a,n}, y_{t-1}^{b,l})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{G,a,n}, z_{t-1}^{C,b',l'}) &= Cov(z_t^{G,a,n}, r_{z^C z^G}^{l} z_{t-1}^{G,b,l} + r_{z^C z^C}^{l} z_{t-1}^{C,b,l} + r_{z^C y}^{l} y_{t-1}^{b,l}) \\
&= r_{z^C z^G}^{l} Cov(z_t^{G,a,n}, z_{t-1}^{G,b,l}) + r_{z^C z^C}^{l} Cov(z_t^{G,a,n}, z_{t-1}^{C,b,l}) + r_{z^C y}^{l} Cov(z_t^{G,a,n}, y_{t-1}^{b,l})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{G,a,n}, y_{t-1}^{b',l'}) &= Cov(z_t^{G,a,n}, r_{yz^G}^{l} z_{t-1}^{G,b,l} + r_{yz^C}^{l} z_{t-1}^{C,b,l} + r_{yy}^{l} y_{t-1}^{b,l}) \\
&= r_{yz^G}^{l} Cov(z_t^{G,a,n}, z_{t-1}^{G,b,l}) + r_{yz^C}^{l} Cov(z_t^{G,a,n}, z_{t-1}^{C,b,l}) + r_{yy}^{l} Cov(z_t^{G,a,n}, y_{t-1}^{b,l})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{C,a,n}, z_{t-1}^{G,b',l'}) &= Cov(z_t^{C,a,n}, r_{z^G z^G}^{l} z_{t-1}^{G,b,l} + r_{z^G z^C}^{l} z_{t-1}^{C,b,l} + r_{z^G y}^{l} y_{t-1}^{b,l}) \\
&= r_{z^G z^G}^{l} Cov(z_t^{C,a,n}, z_{t-1}^{G,b,l}) + r_{z^G z^C}^{l} Cov(z_t^{C,a,n}, z_{t-1}^{C,b,l}) + r_{z^G y}^{l} Cov(z_t^{C,a,n}, y_{t-1}^{b,l})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{C,a,n}, z_{t-1}^{C,b',l'}) &= Cov(z_t^{C,a,n}, r_{z^C z^G}^{l} z_{t-1}^{G,b,l} + r_{z^C z^C}^{l} z_{t-1}^{C,b,l} + r_{z^C y}^{l} y_{t-1}^{b,l}) \\
&= r_{z^C z^G}^{l} Cov(z_t^{C,a,n}, z_{t-1}^{G,b,l}) + r_{z^C z^C}^{l} Cov(z_t^{C,a,n}, z_{t-1}^{C,b,l}) + r_{z^C y}^{l} Cov(z_t^{C,a,n}, y_{t-1}^{b,l})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{C,a,n}, y_{t-1}^{b',l'}) &= Cov(z_t^{C,a,n}, r_{yz^G}^{l} z_{t-1}^{G,b,l} + r_{yz^C}^{l} z_{t-1}^{C,b,l} + r_{yy}^{l} y_{t-1}^{b,l}) \\
&= r_{yz^G}^{l} Cov(z_t^{C,a,n}, z_{t-1}^{G,b,l}) + r_{yz^C}^{l} Cov(z_t^{C,a,n}, z_{t-1}^{C,b,l}) + r_{yy}^{l} Cov(z_t^{C,a,n}, y_{t-1}^{b,l})
\end{aligned}
$$

$$
\begin{aligned}
Cov(y_t^{a,n}, z_{t-1}^{G,b',l'}) &= Cov(y_t^{a,n}, r_{z^G z^G}^{l} z_{t-1}^{G,b,l} + r_{z^G z^C}^{l} z_{t-1}^{C,b,l} + r_{z^G y}^{l} y_{t-1}^{b,l}) \\
&= r_{z^G z^G}^{l} Cov(y_t^{a,n}, z_{t-1}^{G,b,l}) + r_{z^G z^C}^{l} Cov(y_t^{a,n}, z_{t-1}^{C,b,l}) + r_{z^G y}^{l} Cov(y_t^{a,n}, y_{t-1}^{b,l})
\end{aligned}
$$

$$
Cov(y_t^{a,n}, z_{t-1}^{C,b',l'}) = Cov(y_t^{a,n}, r_{z^C z^G}^{l} z_{t-1}^{G,b,l} + r_{z^C z^C}^{l} z_{t-1}^{C,b,l} + r_{z^C y}^{l} y_{t-1}^{b,l})
$$

$$= r^l_{z^C z^G} Cov(y^{a,n}_t, z^{G,b,l}_{t-1}) + r^l_{z^C z^C} Cov(y^{a,n}_t, z^{C,b,l}_{t-1}) + r^l_{z^C y} Cov(y^{a,n}_t, y^{b,l}_{t-1})$$

$$\begin{aligned} Cov(y^{a,n}_t, y^{b',l'}_{t-1}) &= Cov(y^{a,n}_t, r^l_{yz^G} z^{G,b,l}_{t-1} + r^l_{yz^C} z^{C,b,l}_{t-1} + r^l_{yy} y^{b,l}_{t-1}) \\ &= r^l_{yz^G} Cov(y^{a,n}_t, z^{G,b,l}_{t-1}) + r^l_{yz^C} Cov(y^{a,n}_t, z^{C,b,l}_{t-1}) + r^l_{yy} Cov(y^{a,n}_t, y^{b,l}_{t-1}) \end{aligned}$$

where a and b are siblings in law.

Sibling of the sibling in law $a' - c$

We have to compute the covariances between "a'" and "c". Let $n' = m, f$ be the gender of a' and $o = m, f$ the gender of the c. We project a' on his/her sibling b who has gender l

$$\begin{aligned} Cov(z^{G,a',n'}_{t-1}, z^{G,c,o}_{t-1}) &= Cov(r^{n',l}_{z^G z^G} z^{G,b,l}_{t-1} + r^{n',l}_{z^G z^C} z^{C,b,l}_{t-1} + r^{n',l}_{z^G y} y^{b,l}_{t-1}, z^{G,c,o}_{t-1}) \\ &= r^{n',l}_{z^G z^G} Cov(z^{G,b,l}_{t-1}, z^{G,c,o}_{t-1}) + r^{n',l}_{z^G z^C} Cov(z^{C,b,l}_{t-1}, z^{G,c,o}_{t-1}) + r^{n',l}_{z^G y} Cov(y^{b,l}_{t-1}, z^{G,c,o}_{t-1}) \end{aligned}$$

$$\begin{aligned} Cov(z^{G,a',n'}_{t-1}, z^{C,c,o}_{t-1}) &= Cov(r^{n',l}_{z^G z^G} z^{G,b,l}_{t-1} + r^{n',l}_{z^G z^C} z^{C,b,l}_{t-1} + r^{n',l}_{z^G y} y^{b,l}_{t-1}, z^{C,c,o}_{t-1}) \\ &= r^{n',l}_{z^G z^G} Cov(z^{G,b,l}_{t-1}, z^{C,c,o}_{t-1}) + r^{n',l}_{z^G z^C} Cov(z^{C,b,l}_{t-1}, z^{C,c,o}_{t-1}) + r^{n',l}_{z^G y} Cov(y^{b,l}_{t-1}, z^{C,c,o}_{t-1}) \end{aligned}$$

$$\begin{aligned} Cov(z^{G,a',n'}_{t-1}, y^{c,o}_{t-1}) &= Cov(r^{n',l}_{z^G z^G} z^{G,b,l}_{t-1} + r^{n',l}_{z^G z^C} z^{C,b,l}_{t-1} + r^{n',l}_{z^G y} y^{b,l}_{t-1}, y^{c,o}_{t-1}) \\ &= r^{n',l}_{z^G z^G} Cov(z^{G,b,l}_{t-1}, y^{c,o}_{t-1}) + r^{n',l}_{z^G z^C} Cov(z^{C,b,l}_{t-1}, y^{c,o}_{t-1}) + r^{n',l}_{z^G y} Cov(y^{b,l}_{t-1}, y^{c,o}_{t-1}) \end{aligned}$$

$$\begin{aligned} Cov(z^{C,a',n'}_{t-1}, z^{G,c,o}_{t-1}) &= Cov(r^{n',l}_{z^C z^G} z^{G,b,l}_{t-1} + r^{n',l}_{z^C z^C} z^{C,b,l}_{t-1} + r^{n',l}_{z^C y} y^{b,l}_{t-1}, z^{G,c,o}_{t-1}) \\ &= r^{n',l}_{z^C z^G} Cov(z^{G,b,l}_{t-1}, z^{G,c,o}_{t-1}) + r^{n',l}_{z^C z^C} Cov(z^{C,b,l}_{t-1}, z^{G,c,o}_{t-1}) + r^{n',l}_{z^C y} Cov(y^{b,l}_{t-1}, z^{G,c,o}_{t-1}) \end{aligned}$$

$$\begin{aligned} Cov(z^{C,a',n'}_{t-1}, z^{C,c,o}_{t-1}) &= Cov(r^{n',l}_{z^C z^G} z^{G,b,l}_{t-1} + r^{n',l}_{z^C z^C} z^{C,b,l}_{t-1} + r^{n',l}_{z^C y} y^{b,l}_{t-1}, z^{C,c,o}_{t-1}) \\ &= r^{n',l}_{z^C z^G} Cov(z^{G,b,l}_{t-1}, z^{C,c,o}_{t-1}) + r^{n',l}_{z^C z^C} Cov(z^{C,b,l}_{t-1}, z^{C,c,o}_{t-1}) + r^{n',l}_{z^C y} Cov(y^{b,l}_{t-1}, z^{C,c,o}_{t-1}) \end{aligned}$$

$$\begin{aligned} Cov(z^{C,a',n'}_{t-1}, y^{c,o}_{t-1}) &= Cov(r^{n',l}_{z^C z^G} z^{G,b,l}_{t-1} + r^{n',l}_{z^C z^C} z^{C,b,l}_{t-1} + r^{n',l}_{z^C y} y^{b,l}_{t-1}, y^{c,o}_{t-1}) \\ &= r^{n',l}_{z^C z^G} Cov(z^{G,b,l}_{t-1}, y^{c,o}_{t-1}) + r^{n',l}_{z^C z^C} Cov(z^{C,b,l}_{t-1}, y^{c,o}_{t-1}) + r^{n',l}_{z^C y} Cov(y^{b,l}_{t-1}, y^{c,o}_{t-1}) \end{aligned}$$

$$\begin{aligned} Cov(y^{a',n'}_{t-1}, z^{G,c,o}_{t-1}) &= Cov(r^{n',l}_{yz^G} z^{G,b,l}_{t-1} + r^{n',l}_{yz^C} z^{C,b,l}_{t-1} + r^{n',l}_{yy} y^{b,l}_{t-1}, z^{G,c,o}_{t-1}) \\ &= r^{n',l}_{yz^G} Cov(z^{G,b,l}_{t-1}, z^{G,c,o}_{t-1}) + r^{n',l}_{yz^C} Cov(z^{C,b,l}_{t-1}, z^{G,c,o}_{t-1}) + r^{n',l}_{yy} Cov(y^{b,l}_{t-1}, z^{G,c,o}_{t-1}) \end{aligned}$$

$$
\begin{aligned}
Cov(y_{t-1}^{a',n'}, z_{t-1}^{C,c,o}) &= Cov(r_{yz^G}^{n',l} z_{t-1}^{G,b,l} + r_{yz^C}^{n',l} z_{t-1}^{C,b,l} + r_{yy}^{n',l} y_{t-1}^{b,l}, z_{t-1}^{C,c,o}) \\
&= r_{yz^G}^{n',l} Cov(z_{t-1}^{G,b,l}, z_{t-1}^{C,c,o}) + r_{yz^C}^{n',l} Cov(z_{t-1}^{C,b,l}, z_{t-1}^{C,c,o}) + r_{yy}^{n',l} Cov(y_{t-1}^{b,l}, z_{t-1}^{C,c,o})
\end{aligned}
$$

$$
\begin{aligned}
Cov(y_{t-1}^{a',n'}, y_{t-1}^{c,o}) &= Cov(r_{yz^G}^{n',l} z_{t-1}^{G,b,l} + r_{yz^C}^{n',l} z_{t-1}^{C,b,l} + r_{yy}^{n',l} y_{t-1}^{b,l}, y_{t-1}^{c,o}) \\
&= r_{yz^G}^{n',l} Cov(z_{t-1}^{G,b,l}, y_{t-1}^{c,o}) + r_{yz^C}^{n',l} Cov(z_{t-1}^{C,b,l}, y_{t-1}^{c,o}) + r_{yy}^{n',l} Cov(y_{t-1}^{b,l}, y_{t-1}^{c,o})
\end{aligned}
$$

where b and c are siblings in law.

We can recursively compute the covariances for siblings in law, spouses of the sibling and siblings of the siblings of any degree.

Cousins in law $aa - cc$

We have to compute the covariances between "aa" and "cc". Let $n^* = m, f$ be the gender of aa and $o^* = m, f$ the gender of the cc. We project cc on c (his/her father or mother) who has gender o

$$
\begin{aligned}
Cov(z_t^{G,aa,n^*}, z_t^{G,cc,o^*}) &= Cov(z_t^{G,aa,n^*}, G_{zgo}^{o^*} z_{t-1}^{G,b,o} + G_{zo}^{o^*} z_{t-1}^{C,b,o} + G_{yo}^{o^*} y_{t-1}^{b,o}) \\
&= G_{zgo}^{o^*} Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b,o}) + G_{zo}^{o^*} Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b,o}) + G_{yo}^{o^*} Cov(z_t^{G,aa,n^*}, y_{t-1}^{b,o})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{G,aa,n^*}, z_t^{C,cc,o^*}) &= Cov(z_t^{G,aa,n^*}, C_{zgo}^{o^*} z_{t-1}^{G,b,o} + C_{zo}^{o^*} z_{t-1}^{C,b,o} + C_{yo}^{o^*} y_{t-1}^{b,o}) \\
&= C_{zgo}^{o^*} Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b,o}) + C_{zo}^{o^*} Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b,o}) + C_{yo}^{o^*} Cov(z_t^{G,aa,n^*}, y_{t-1}^{b,o})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{G,aa,n^*}, y_t^{cc,o^*}) &= Cov(z_t^{G,aa,n^*}, B_{zgo}^{o^*} z_{t-1}^{G,b,o} + B_{zo}^{o^*} z_{t-1}^{C,b,o} + B_{yo}^{o^*} y_{t-1}^{b,o}) \\
&= B_{zgo}^{o^*} Cov(z_t^{G,aa,n^*}, z_{t-1}^{G,b,o}) + B_{zo}^{o^*} Cov(z_t^{G,aa,n^*}, z_{t-1}^{C,b,o}) + B_{yo}^{o^*} Cov(z_t^{G,aa,n^*}, y_{t-1}^{b,o})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{C,aa,n^*}, z_t^{G,cc,o^*}) &= Cov(z_t^{C,aa,n^*}, G_{zgo}^{o^*} z_{t-1}^{G,b,o} + G_{zo}^{o^*} z_{t-1}^{C,b,o} + G_{yo}^{o^*} y_{t-1}^{b,o}) \\
&= G_{zgo}^{o^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b,o}) + G_{zo}^{o^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b,o}) + G_{yo}^{o^*} Cov(z_t^{C,aa,n^*}, y_{t-1}^{b,o})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{C,aa,n^*}, z_t^{C,cc,o^*}) &= Cov(z_t^{C,aa,n^*}, C_{zgo}^{o^*} z_{t-1}^{G,b,o} + C_{zo}^{o^*} z_{t-1}^{C,b,o} + C_{yo}^{o^*} y_{t-1}^{b,o}) \\
&= C_{zgo}^{o^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b,o}) + C_{zo}^{o^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b,o}) + C_{yo}^{o^*} Cov(z_t^{C,aa,n^*}, y_{t-1}^{b,o})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{C,aa,n^*}, y_t^{cc,o^*}) &= Cov(z_t^{C,aa,n^*}, B_{zgo}^{o^*} z_{t-1}^{G,b,o} + B_{zo}^{o^*} z_{t-1}^{C,b,o} + B_{yo}^{o^*} y_{t-1}^{b,o}) \\
&= B_{zgo}^{o^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{G,b,o}) + B_{zo}^{o^*} Cov(z_t^{C,aa,n^*}, z_{t-1}^{C,b,o}) + B_{yo}^{o^*} Cov(z_t^{C,aa,n^*}, y_{t-1}^{b,o})
\end{aligned}
$$

$$
\begin{aligned}
Cov(y_t^{aa,n^*}, z_t^{G,cc,o^*}) &= Cov(y_t^{aa,n^*}, G_{zgo}^{o^*} z_{t-1}^{G,b,o} + G_{zo}^{o^*} z_{t-1}^{C,b,o} + G_{yo}^{o^*} y_{t-1}^{b,o}) \\
&= G_{zgo}^{o^*} Cov(y_t^{aa,n^*}, z_{t-1}^{G,b,o}) + G_{zo}^{o^*} Cov(y_t^{aa,n^*}, z_{t-1}^{C,b,o}) + G_{yo}^{o^*} Cov(y_t^{aa,n^*}, y_{t-1}^{b,o})
\end{aligned}
$$

$$
\begin{aligned}
Cov(z_t^{G,aa,n^*}, z_t^{C,cc,o^*}) &= Cov(y_t^{aa,n^*}, C_{zgo}^{o^*} z_{t-1}^{G,b,o} + C_{zo}^{o^*} z_{t-1}^{C,b,o} + C_{yo}^{o^*} y_{t-1}^{b,o}) \\
&= C_{zgo}^{o^*} Cov(y_t^{aa,n^*}, z_{t-1}^{G,b,o}) + C_{zo}^{o^*} Cov(y_t^{aa,n^*}, z_{t-1}^{C,b,o}) + C_{yo}^{o^*} Cov(y_t^{aa,n^*}, y_{t-1}^{b,o})
\end{aligned}
$$

$$
\begin{aligned}
Cov(y_t^{aa,n^*}, y_t^{cc,o^*}) &= Cov(y_t^{aa,n^*}, B_{zgo}^{o^*} z_{t-1}^{G,b,o} + B_{zo}^{o^*} z_{t-1}^{C,b,o} + B_{yo}^{o^*} y_{t-1}^{b,o}) \\
&= B_{zgo}^{o^*} Cov(y_t^{aa,n^*}, z_{t-1}^{G,b,o}) + B_{zo}^{o^*} Cov(y_t^{aa,n^*}, z_{t-1}^{C,b,o}) + B_{yo}^{o^*} Cov(y_t^{aa,n^*}, y_{t-1}^{b,o})
\end{aligned}
$$

where c is the sibling in law of the uncle/aunt of aa. We can recursively compute the covariances for cousins in law of any degree.